ELEMENTARY CRYSTALLOGRAPHY

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PREFACE

This book is intended to meet the need for an elementary text book on Crystallography. It is founded on the instruction in that subject which has been given for some years past at Birkbeck College to students preparing for the Intermediate Examination of the University of London. It has been our object to make the principles of Crystallography intelligible to those who have only a moderate mathematical training, and especially to afford such information as will assist in the recognition of mineral species. While written primarily for the student of Geology and Mineralogy, the book should also prove useful to the Chemist and Physicist requiring an introduction to the study of crystals.

It is hoped that the practical exercises appended to each chapter will materially assist in familiarising the student with the subject.

The illustrations, with a few exceptions, have been specially drawn for this book by Mrs. G. M. Davies. We are indebted to Mrs. S. L. Penfield for kind permission to reproduce *Fig.* 31, to Messrs. J. H. Steward, Ltd., for the block of *Fig.* 32, and to the Council of the Mineralogical Society for the blocks of *Figs.* 157 to 175.

The limits assigned to the book did not allow of the inclusion of the details of stereographic and gnomonic pro-

jection, or crystallographic calculations. This is the less to be regretted, as the work by Mr. T. V. Barker on "Graphical and Tabular Methods in Crystallography" is now available. The advanced student is also referred to the well-known treatises on crystallography by Professor Lewis and Dr. Tutton.

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ELEMENTARY CRYSTALLOGRAPHY

CHAPTER I.

INTRODUCTION.

To most people a crystal is a solid substance bounded by plane faces with definite geometrical shapes, such as triangles, parallelograms or trapezoids. This outer form is undoubtedly the most striking character of the majority of crystals, but it is not the essential feature that distinguishes a crystalline substance from others. An exact model of a crystal made in glass remains glass, and is not a crystal, while a crystal of quartz ground down to a spherical form is still a crystal, although it has no plane faces. The form, when it appears, is only the external evidence of the internal molecular structure, and it is the regular arrangement of the molecules, or rather of the atoms, that is the real criterion of a crystal. Even liquid crystals are known.

The term c r y s t a l is derived from the Greek word for ice. It was applied originally to the water-clear quartz, or rock-crystal, of the Alps, in the belief that this was really ice that had been subjected to such intense and long-continued cold that it could not melt. Afterwards the term was extended to other minerals occurring in regular forms. Vessels and ornaments were at one time carved out of rock-crystal, and when glass was substituted the ware was still known as crystal, although this use of the term is inadmissible from the scientific point of view, since glass has not a crystalline structure.

By means of the interference phenomena of X-rays, first observed by Professor Laue and subsequently studied by Sir William Bragg, Professor W. L. Bragg, and others, it has been possible to determine in many cases the exact positions of the atoms of a crystal relatively to one another, and to

show that, as already supposed, they are arranged with the utmost regularity. Just as in wall-paper, where the same design is continually repeated, corresponding points in the pattern can be seen to lie in innumerable parallel rows in different directions, intersecting one another, so the atoms of a crystal are arranged in a multitude of straight rows which cross one another in a regular pattern. The rows themselves may be combined in more ways than one to form a vast number of atomic planes, or, as they are usually termed, atomic nets, and these nets intersect one another in a regular manner in straight lines, which are themselves rows of atoms.

The whole assemblage of rows and nets constitutes what is known as a space lattice or simply lattice, which is in fact another name for the crystal structure as a whole in three dimensions.

The faces of a crystal are, as might be expected, parallel to nets of atoms in the crystal structure, and so are the crystal cleavages; that is to say the planes along which most crystals have a tendency to split if a sufficient shearing force is applied to them.

In the same manner the edges in which crystal faces meet are parallel to rows of atoms in which nets of atoms intersect.

In amorphous or non-crystalline substances, on the other hand, including all gases, the vast majority of liquids and certain solids, such as glass, the atoms occur in small more or less independent groups known as molecules, each consisting of a very limited number of atoms united by bonds, but when a substance passes into the crystalline state the molecules in many cases lose their identity for the time being. In a solution of sodium chloride each atom of sodium is supposed to be united to one of chlorine, but in crystalline sodium chloride, which can be formed from such a solution by the evaporation of the water, each atom of sodium is (except, of course, on the surface of the crystal) united to six atoms of chlorine, and each atom of chlorine to six of sodium, the whole forming a continuous rectangular

structure made up of cubic cells. In this structure each cube contains four sodiums and four chlorines, arranged so that adjoining atoms belong to different elements, while each diagonal of a cube-face has at its ends atoms of the same element. The edges of a succession of adjoining cubic cells constitute an atomic row of alternate sodium and chlorine atoms (see *Fig.* 1), whereas a row made up of a succession of diagonals of the cube-faces will consist entirely of sodium atoms or entirely of chlorine atoms.

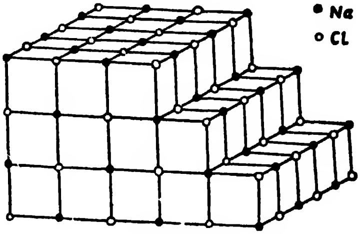

FIG. 1.

Most crystal structures consist of six-sided cells like those of sodium chloride, but they are by no means always cubes, for in many cases the edges of the cells do not meet at right angles, and as a rule those parallel to different directions are unequal in length.

When crystallisation occurs the atoms of the molecules take up their places in the crystal structure just as soldiers in small squads may form up in a battalion.

In some cases, however, especially in the simpler organic compounds, the arrangement of the atoms in a cell appears to bear a close relation to that in the molecule.

If the process of crystallisation be slow large crystals are built up, but the more rapidly it takes place the smaller are the crystals, as there is no time for the molecules to gather together to form larger regular structures. If the transition to the solid state is very rapid, there is frequently no evidence of the existence of crystalline structure, and the substance is regarded as non-crystalline or a m o r p h o u s.*

The same molten material which would form, if cooled sufficiently slowly, a granite composed of comparatively

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* In some cases, however, the action of some apparently non-crystalline substances on the X-rays gives us reason to believe that ultra-microscopic crystals are present.

large crystals of quartz, felspar and mica* may, if cooled rapidly at the earth's surface or by contact with cold rocks, solidify as a volcanic glass.

A clear distinction must accordingly be drawn between crystallisation and solidification, though the two frequently constitute a single operation. A non-crystalline substance like glue or glass may be regarded as an extremely rigid liquid, though it is a solid in the ordinary acceptation of the term. Such a non-crystalline substance may ultimately pass into the crystalline state by a slow readjustment of the atoms. This process is known as devitrification.

The same substance may develop different crystal structures according to the conditions under which it crystallises. Such a substance is said to be dimorphic or polymorphic.

Crystals may even come into existence by a rearrangement of the atoms of matter which is already crystallised after another fashion.

Substances whose genesis is the result of chemical reactions will also appear in the crystalline state, provided the product is formed sufficiently slowly.

If a saturated solution of alum in water is allowed to evaporate slowly, the alum separates as crystals which have eight faces. When equally developed these faces are equilateral triangles, and the form (*Fig.* 2) is known as an octahedron. Frequently, however, some of the faces are larger than the others (*Figs.* 3 and 4), and the crystal may also be interfered with by adjacent crystals, and then only the constancy of the angles between the faces assures us that these ill-formed crystals are really octahedra.

Crystals formed under human control are often called artificial crystals to distinguish them from the natural crystals found in the rocks. The formation of these crystals is, however, the natural result of the conditions; we provide suitable conditions, Nature does the rest.

* The presence of a certain amount of the elements of water is also necessary if a granite is to form.

If the growing crystals are watched, it will be seen that matter is constantly added to their surface in such a way that the new surface thus formed is parallel to the old one. Thus each face of a large crystal of alum is essentially the same face as when the crystal was minute, and its inclination to the other faces is unaltered. Growth ceases when the supply of fresh alum ceases, but is renewed when the conditions are again favourable.

In crystals it is not the size of the faces that is important, since that varies as the crystal grows, nor their shape, since that depends on the development of the adjoining faces. The really essential point is, as we have seen, the *angle between the faces*. In 1669 Nicolaus Steno found that the angles between corresponding faces of quartz crystals are always the same, however irregular the

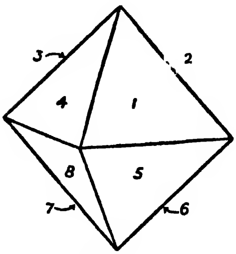


FIG. 2.

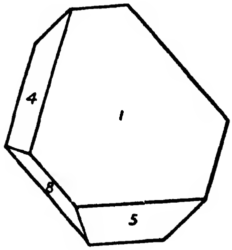


FIG. 3.

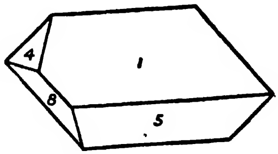


FIG. 4.

crystals might be in the development of faces, and a similar uniformity of angles has since been found to be general among crystals. This *law of the constancy of angles* in all crystals of the same mineral or substance is the fundamental law of crystallography. The eye is far less able to recognise equality of inclination than similarity of shape; on this account it is usual in models or drawings of crystals to represent similar edges as equal in length, and so to make the symmetry of the crystal as obvious as possible. These models or drawings may be said to represent the ideal forms that the crystals would attain if not prevented by external circumstances, such as irregular

supply of material for growth or the proximity of other solid bodies. When he is familiar with these ideal forms the student will be able to recognise them even when imperfectly developed in actual crystals, and will find that crystallography is not a useless branch of mathematics, but a valuable aid in the identification of minerals.

CHAPTER II.

SYMMETRY.

PLANES OF SYMMETRY.

Among animals and plants various types of symmetry are seen, the most familiar being the *bilateral* (*i.e.*, two-sided) symmetry shown by most vertebrates. Generally speaking the limbs on the right side correspond in size with those on the left, and if a bird could be divided in two by a vertical plane passing through the middle of the beak and tail, and one half placed against a mirror, the image reflected in the mirror would appear to replace the missing half. That is an example of *symmetry about a plane*, and, as there is only one way in which a bird could be divided to give the same result, the bird has only one *plane of symmetry*. In perfect crystals, and in crystal models, the same test may be applied, the mirror-image of one half in a plane of symmetry coinciding with the other half. To state this relation in other words, if from any point on the crystal a perpendicular line, or normal, be drawn to a plane of symmetry, a similar point will be found on the same line on the opposite side of the plane and at the same distance from it.

On account of the imperfection of most natural crystals, however, it may be better to define a plane of symmetry as a plane which has on opposite sides of it a similar distribution of similar points (and hence of similar edges and faces).

Thus in *Figs.* 5 and 6, where the lines $D\bar{B}$, $A\bar{C}$ and $B\bar{D}$ are vertical, DB is horizontal, and CA slopes down from C to A, there is a plane of symmetry passing through the points $C A \bar{C} \bar{A}$, but the plane that passes through $D B \bar{D} \bar{B}$ is not a plane of symmetry since C is not exactly opposite to A.

In *Fig.* 7, where the upper and lower faces are squares and horizontal, there are five planes of symmetry, four vertical and one horizontal. Two of the vertical planes of symmetry, shown by interrupted lines - - - except where they coincide with the edges of the crystal, pass diagonally across the horizontal faces and through opposite vertical edges; and two, shown by dots . . . . . ., are parallel to the vertical faces and midway between them, and therefore cut the horizontal faces parallel to their edges. The horizontal plane of symmetry, also shown by dots, is in like manner parallel to the horizontal faces and midway between them.

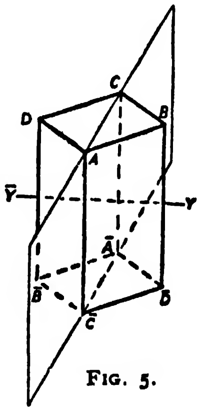


FIG. 5.

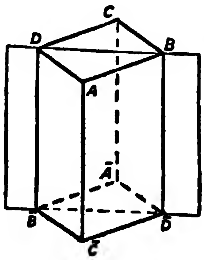


FIG. 6.

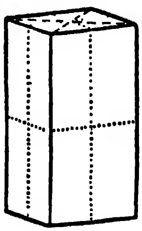


FIG. 7.

In the cube (*Fig.* 8) there are nine planes of symmetry. Three of these, shown by dots, are parallel to the faces of the cube, and six, shown by interrupted lines, pass through the cube edges and diagonally across the cube faces. These planes of symmetry are shown more clearly on page 71, *Figs.* 94 and 95.

In the octahedron also (*Fig.* 9) there are nine planes of symmetry. Three of these pass through the crystal edges and six bisect opposite edges at right angles, and pass through opposite corners, or c o i g n s as they are usually called in crystallography from the old spelling of the French word for a corner.

LINES OF SYMMETRY.

Crystals may also exhibit *symmetry about a line*. In ideal crystals a *line of symmetry* is such that every point on the crystal has a similar point at the same distance on the opposite side of the line of symmetry, and lying on the same normal to it.

In *Fig.* 5 the horizontal line $Y\bar{Y}$ through the middle points of $B\bar{D}$ and $D\bar{B}$ is a line of symmetry. In *Fig.* 7 there are five and in *Figs.* 8 and 9 there are nine lines of symmetry.

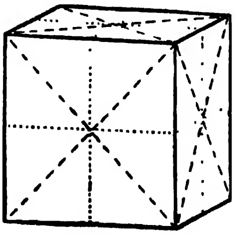


FIG. 8.

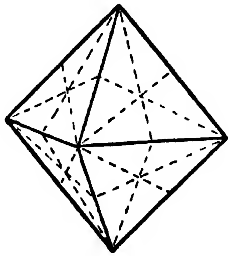


FIG. 9.

CENTRE OF SYMMETRY.

Many crystals have also *symmetry about a point*. Such a point, or *centre of symmetry*, has the property that all straight lines that can be drawn through it will pass through a pair of similar points lying on opposite sides of the centre of symmetry, and at the same distance from it. It necessarily follows from this that all faces and edges will occur in parallel pairs on opposite sides of a centre of symmetry; consequently, if a perfectly developed crystal having a centre of symmetry is laid on a horizontal table on any face, there will be a similar horizontal face above. If, on the other hand, a perfect crystal or model can be laid with one face flat on the table and no corresponding horizontal face above, the faces are not all in parallel pairs, and there is no centre of symmetry. In *Figs.* 5 to 9 there is a centre of symmetry. In *Fig.* 5 the

point B corresponds to the point $\bar{B}$ on the opposite side of the centre, the edge BC to the edge $\bar{B}\bar{C}$, the face ABCD to the face $\bar{A}\bar{B}\bar{C}\bar{D}$, and so on. In the tetrahedron (*Fig.* 10), on the other hand, there is no centre of symmetry, and if such a crystal is laid with one face flat on the table, a coign and not a horizontal face will be found uppermost. Some crystals of quartz have all their faces in parallel pairs (*Fig.* 156), but others have the faces of the trigonal pyramid and trapezohedron (*Figs.* 151-152, p. 109), which are not paired in this way. Quartz has therefore no centre of symmetry.

Turning again to *Fig.* 5 we may compare the results of the three kinds of symmetry that it illustrates, due to (1) the plane of symmetry $CA\bar{C}\bar{A}$, (2) the line of symmetry $Y\bar{Y}$, and (3) the centre of symmetry. These are shown in the following table :—

	Plane of symmetry, $CA\bar{C}\bar{A}$.	Line of symmetry, $Y\bar{Y}$.	Centre of symmetry.
Pairs of points.	B, D	A, $\bar{A}$	A, $\bar{A}$
	$\bar{D}$, $\bar{B}$	B $\bar{D}$	B, $\bar{B}$
		C, $\bar{C}$	C, $\bar{C}$
		D, $\bar{B}$	D, $\bar{D}$
Pairs of edges.	AB, AD	AB, $\bar{A}\bar{D}$	AB, $\bar{A}\bar{B}$
	BC, CD	BC, $\bar{C}\bar{D}$	BC, $\bar{B}\bar{C}$
	$\bar{C}\bar{D}$, $\bar{B}\bar{C}$	CD, $\bar{B}\bar{C}$	CD, $\bar{C}\bar{D}$
	$\bar{A}\bar{D}$, $\bar{A}\bar{B}$	DA, $\bar{A}\bar{B}$	DA, $\bar{D}\bar{A}$
	$B\bar{D}$, $\bar{B}D$	$A\bar{C}$, $\bar{A}C$	$A\bar{C}$, $\bar{A}C$
			$B\bar{D}$, $\bar{B}D$
Pairs of faces.	$AB\bar{D}\bar{C}$, $AD\bar{B}\bar{C}$	ABCD, $\bar{A}\bar{B}\bar{C}\bar{D}$	ABCD, $\bar{A}\bar{B}\bar{C}\bar{D}$
	$CB\bar{D}\bar{A}$, $CD\bar{B}\bar{A}$	$AB\bar{D}\bar{C}$, $CB\bar{D}\bar{A}$	$AB\bar{D}\bar{C}$, $\bar{A}\bar{B}DC$
		$AD\bar{B}\bar{C}$, $CD\bar{B}\bar{A}$	$CB\bar{D}\bar{A}$, $\bar{C}\bar{B}DA$
	Points on the plane of symmetry and edges and faces at right angles to it are not repeated.	Points on the line of symmetry, edges meeting it at right angles, and faces at right angles to it are not repeated.	When there is a centre of symmetry, all points, edges and faces are repeated.

The human form has usually one plane of approximate symmetry, but no line or centre of symmetry. A line of symmetry through the hips would give a second face where the heels should be. So also would a centre of symmetry. If, however, the plane of symmetry is wanting, through the eyes not matching, for example, a line of symmetry would reproduce the right eye in the position of the right heel, while a centre of symmetry would reproduce it in the position of the left heel. But if right and left are indistinguishable, we see that a centre of symmetry combined with a

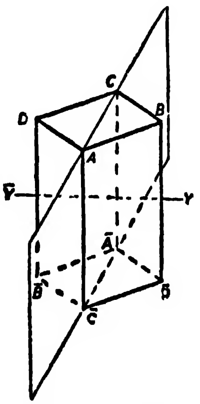


FIG. 5 (*repeated*).

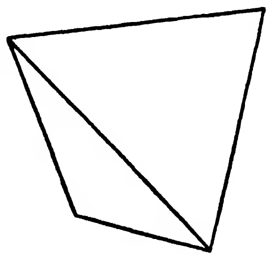


FIG. 10.

plane of symmetry would involve a line of symmetry also. This illustrates a rule which should be remembered: that in all cases where a centre of symmetry is present, there is a line of symmetry at right angles to every plane of symmetry, and *vice versa*, so that the number of planes and lines of symmetry is the same.

Thus, in *Fig.* 5, where there is a centre of symmetry, there is one plane of symmetry and one line of symmetry at right angles to it. In *Figs.* 7, 8 and 9 there is also a centre of symmetry, and in *Fig.* 7 there are five planes of symmetry and five lines of symmetry at right angles to them respectively, while in *Fig.* 8 and in *Fig.* 9 there are nine

planes of symmetry and a line of symmetry at right angles to each of them.

Where there is no centre of symmetry this relation between the planes and lines of symmetry no longer exists. In the tetrahedron, *Fig.* 10, for example, there are six planes of symmetry and only three lines of symmetry.

AXES OF SYMMETRY.

If a crystal be rotated through a half turn (180°) about a line of symmetry, all the faces and edges will occupy the same position as the similar but opposite faces and edges did at first. Thus, if the crystal in *Fig.* 11 be rotated through half a circle about the axis $Y\bar{Y}$, which is a

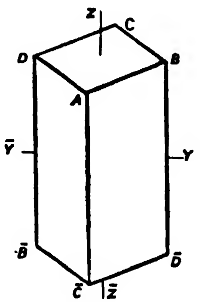


FIG. 11.

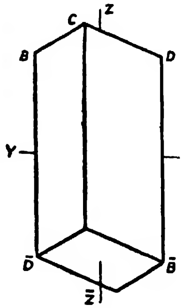


FIG. 12.

line of symmetry, the whole appearance will be the same as before. The line $Z\bar{Z}$, on the other hand, is not a line of symmetry, and half a rotation about it gives the result indicated in *Fig.* 12, which is not identical with the original configuration.

If the half turn is the smallest angle through which a crystal can be rotated about a line of symmetry and reproduce the original configuration, the line of symmetry is said to be a half-turn or digonal axis of symmetry, and have a cyclic number ii. This is the case with the line $Y\bar{Y}$ in *Figs.* 5, 11 and 12, and also the lines distinguished by the Roman numeral ii. in *Figs.* 13, 14, 15 and 16.

If, however, a quarter turn about a line will be sufficient to produce the same result—for instance, a rotation through 90° on any of the lines marked iv. in *Figs.* 13 and 14—such a line is said to be a quarter-turn or tetragonal axis of symmetry, with a cyclic number iv.

If a sixth of a complete rotation about a line will give a like result, for instance, a rotation through 60° about the line vi. in *Fig.* 15, that line is said to be a one-sixth-turn or hexagonal axis of symmetry, with a cyclic number vi.

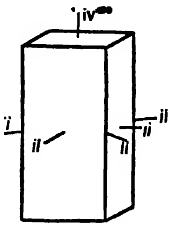


FIG. 13.

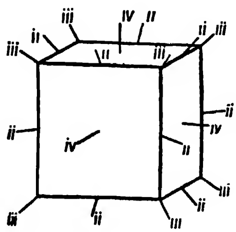


FIG. 14.

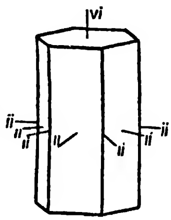


FIG. 15.

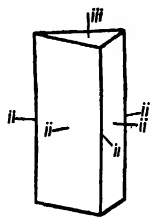


FIG. 16.

It is obvious that not only is every half-turn or digonal axis of symmetry a line of symmetry, but also every quarter-turn or tetragonal axis and one-sixth-turn or hexagonal axis.

Some crystals present an unaltered appearance after a rotation about a line through a third of a circle—for example, a rotation through 120° about the lines marked iii. in *Figs.* 14 and 16. Such a line is a one-third-turn or trigonal axis, and has a cyclic number iii., but it is not a line of symmetry, as it has not similar faces, edges and points on opposite sides of it.

In *Fig.* 13 there is only one quarter-turn axis, but there are four half-turn axes, two passing through the middle points of the two pairs of opposite vertical edges and two through the middle points of the vertical faces. There are thus five lines of symmetry, and this number agrees, as we have seen, with the number of planes of symmetry in the same crystal shown in *Fig.* 7.

In the cube (*Fig.* 14) there are three quarter-turn axes, one passing through the middle points of each of the three pairs of opposite faces. The half-turn axes pass each through the middle points of two of the cube edges, and as there are twelve of these edges there are six half-turn axes. The lines of symmetry are thus $3+6=9$, corresponding to the nine planes of symmetry shown in *Fig.* 8. The trigonal axes are four in number, each passing through two of the eight coigns of the cube.

Fig. 15 has one one-sixth-turn axis and six horizontal half-turn axes, three passing through the middle points of the vertical edges and three through the centres of the vertical faces. There are, therefore, seven lines and seven planes of symmetry.

Fig. 16 has, in addition to the one-third-turn axis, three half-turn axes and four planes of symmetry. A centre of symmetry is present in *Figs.* 13, 14 and 15, and hence these crystals have as many lines as planes of symmetry in accordance with the rule stated on p. 11, but this is not the case with *Fig.* 16, which has no centre of symmetry.

An axis of symmetry with cyclic number v., which is so common in flowers and echinoderms, is never seen in crystals. Indeed, the laws of crystallography which can be proved to be the necessary consequences of the theory, now completely established, that crystals are built up of atoms regularly arranged in rows and nets, are inconsistent with any axial symmetry based on a cyclic number other than two, three, four or six.

In such a crystal as *Fig.* 17, which has eight vertical faces and edges apparently identical with one another, the vertical axis may at first sight be mistaken for one of one-eighth-turn symmetry, but closer observation shows that of the eight vertical edges four are more obtuse than the other four, which alternate with them. The axis is therefore one of quarter-turn symmetry only.

CLASSES OF SYMMETRY.

It can be shown that all the possible combinations of the different types of symmetry occurring in crystals fall into the *thirty-two classes* shown in the table on pp. 126-127. Accordingly every crystal belongs to one or other of these thirty-two classes of symmetry. Two of them have not yet been observed in either natural or artificial crystals, and six more are not known to occur among minerals, while about half the remainder are comparatively rarely seen. Beginners may ignore all but the eleven classes given in the table on p. 16.

It will be seen that the classes are arranged under six or

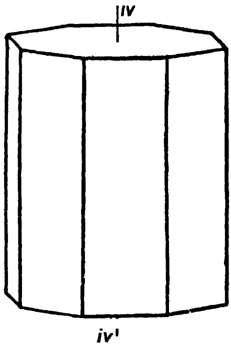


FIG. 17.

seven systems. The classes are determined by the symmetry, as explained above, but the systems are founded mainly upon the crystallographic axes in a manner that will be explained in the next chapter. For the present purpose, however, two of the systems, the hexagonal and trigonal, are placed together, thus reducing the number of systems to six.

A larger table showing all the thirty-two classes, with symbols and names expressing the symmetry by which they are distinguished, will be found on pp. 126-127.

The classes shown in the first column of the following

table are named for the convenience of elementary students after minerals possessing the symmetry of the different classes. Common rock-forming minerals have as far as possible been selected. In the second column the presence of a centre of symmetry is indicated by the letter c. The third column shows the number of planes of symmetry, and the remaining columns the number of axes of digonal, tetragonal, hexagonal and trigonal symmetry. The symbols in the first column are explained in Chapter XV.

THE MOST IMPORTANT CLASSES OF SYMMETRY.

Systems, Classes and Symbols.		Centre of symmetry	Planes of symmetry	Axes of symmetry.			
				ii	iv	vi	iii
CUBIC SYSTEM.							
Spinel Class	CDc	c	9	6	3	—	4
Tetrahedrite Class	CDu	—	6	3	—	—	4
Pyrite Class	CMc	c	3	3	—	—	4
HEXAGONAL SYSTEM.							
Beryl Class	VIDc	c	7	6	—	1	—
Calcite Class	IIIDc	c	3	3	—	—	1
Tourmaline Class	IIIDu	—	3	—	—	—	1
Quartz Class	IIIMh	—	—	3	—	—	1
TETRAGONAL SYSTEM.							
Zircon Class	IVDc	c	5	4	1	—	—
ORTHORHOMBIC SYSTEM.							
Olivine Class	IIDc	c	3	3	—	—	—
MONOCLINIC SYSTEM.							
Augite Class	IIMc	c	1	1	—	—	—
TRICLINIC SYSTEM.							
Albite Class	IMc	c	—	—	—	—	—

PRACTICAL WORK.

Examine a number of crystal models, ascertain the symmetry to be found in each, and hence by means of the foregoing table determine the class to which it belongs.

CHAPTER III.

CRYSTALLOGRAPHIC AXES.

The direction of a straight line may be defined by stating the lengths of the intercepts it makes, that is to say, the lengths it cuts off or would cut off if produced, on two intersecting lines or axes in whose plane it lies. Thus in *Fig.* 18, $X O \bar{X}$ and $Y O \bar{Y}$ are two axes intersecting at a point O, which is called the origin. The line AB has intercepts OA on the axis OX and OE on the axis OY. These intercepts measure 1 cm. and 2 cm. respectively, and they

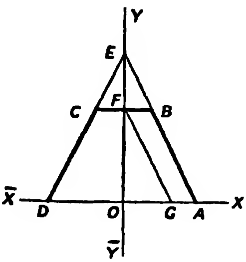


FIG. 18.

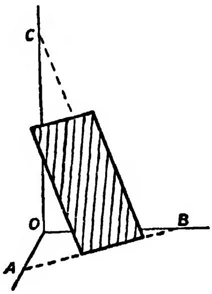


FIG. 19.

serve to fix the position of AB relative to the axes. If we are concerned only with the direction of AB and not its actual position, it will be sufficient to say that its intercepts on OX and OY are as 1 : 2, since all parallel lines will have intercepts in the same ratio. Thus, if GF is parallel to AB, then OA : OE = OG : OF.

Similarly, if we call distances along OX and OY positive, and those along $O\bar{X}$ and $O\bar{Y}$ negative, the intercepts of CD are as $(-1) : 2$. The minus sign is more con-

veniently written above the figure, thus $\bar{1} : 2$. The line CB is parallel to OX, and therefore only meets it at infinity; its intercepts are therefore as $\infty : 1$, where ∞ represents infinity.

In like manner the inclination of a plane may be defined by giving the ratio of its intercepts on three axes, which pass through the same point, the origin, but do not lie in the same plane. Thus, if a drawing-board rests against two walls in the corner of a room, we can describe its slope by giving the distance from the corner at which its plane would (if produced) cut the three lines in which the walls meet the floor and each other (OA, OB and OC in *Fig.* 19). A postcard placed in the same corner parallel to the drawing-board would have different intercepts, oa, ob, oc, but they would be in the same ratio as those of the board; that is, OA : OB : OC = oa : ob : oc. Postcard and drawing-board may be taken to represent the same face of a crystal at different stages of its growth.

Although a plane may be defined by referring it to any three axes, provided they are not all in one plane, there are in crystals certain directions which yield more simple results than others when chosen as axes. It is found that this object is attained if the axes chosen are parallel to edges, or possible edges* of the crystal. The axial plane in which any two axes lie will then be parallel to a face, or a possible face,* of the crystal. Where there are several sets of edges that would give equally good results, it is customary to follow the crystallographer who first described the mineral and use the axes he selected.

Crystallographic axes are, therefore, to a large extent conventional, but they probably give the best results when they correspond to the edges of the ultimate cells of which the crystal structure is made up, so that the axial planes correspond to the faces of those cells. The axial planes are

---

* By possible edges and faces are meant those that might exist consistently with the laws of crystallography in association with the actual edges and faces observed. See p. 30.

then, it is believed, parallel to the nets in which the atomic density is greatest; that is to say, in which there is the maximum number of atoms to the unit area, and such nets again are those which are at the greatest distance from one another. At present, however, these data are still imperfectly known.

The order in which the axes are taken is also a matter of convention. It is customary to hold the crystal so that one axis points towards the observer, one is right and left (or approximately so), and the third vertical, and to consider them in that order. The axes are conveniently referred to as OA, OB and OC, or the axes of *a*, *b* and *c* (*Fig.* 20).

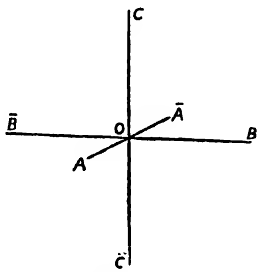


FIG. 20.

The positive direction of the axis OA is in front,
The positive direction of the axis OB is to the right,
The positive direction of the axis OC is on top;
The negative direction of the axis OA is at the back,
The negative direction of the axis OB is to the left,
The negative direction of the axis OC is underneath.

It will be seen that the initials of front, right, top, and of back, left, underneath, are in alphabetical order, and thus form a mnemonic for the order of the axes.

Although this order was adopted by Weiss, and is followed by most modern crystallographers, there are crystals both at the Natural History Museum and at the Museum of Practical Geology in London, the faces of which are marked in conformity with Miller's arrangement, in

which the axes OA and OB are transposed. These may prove confusing to students who are not aware of the change of axes.

By their intersection at the point O the axial planes form eight corners or trihedral angles, known as octants (see *Fig.* 43, p. 42). The front, right, top octant is bounded by OA, OB and OC; the front, left, top octant by OA, $O\bar{B}$ and OC; the back, right, top octant by $O\bar{A}$, OB and OC; and so on.

In a crystal of olivine the axes OA, OB and OC are all at right angles. Careful measurement of the angles in olivine shows that some of the faces present meet the axes with intercepts in the following ratios (*Fig.* 21):—

	Axis OA		Axis OB		Axis OC
Face *a*,*	1	:	∞	:	∞
" *m*,	0.46575	:	1	:	∞
" *s*,	0.93150	:	1	:	∞
" *r*,	1.39725	:	1	:	∞
" *k*,	∞	:	1	:	1.17302
" *e*,	0.46575	:	1	:	0.58651
" *o*,	0.46575	:	1	:	0.29325
" *f*,	0.93150	:	1	:	1.17302

It is evident that the face *a*, which meets axes OB and OC at infinity, must be parallel to these two axes, and that the faces *m*, *s* and *r* are all parallel to the vertical axis OC and cut the axes OA and OB at different distances.

It will be seen that the ratios can only be expressed in numerous decimals. The greater the accuracy of the observations the greater will be the number of decimals required to express the result. A careful examination of these ratios shows, however, that we can avoid the repetition of these cumbersome decimals in expressing the ratios of the inter-

* The letters *a, m, s, r, k, e, o, f*, etc., are conventionally attached to faces, in particular minerals, but, as a rule, according to no recognised principle.

cepts by taking a face which meets all three axes as our standard or unit face and writing the intercepts of all other faces as multiples or submultiples of its intercepts. In the case of olivine the face *e* has been selected by custom as this standard; it is known as the unit or parametral plane (Gr. *para*, beside or along, *metron*, measure), and its intercepts, 0.46575 : 1 : 0.58651, are termed the parameters of olivine. They are usually denoted by

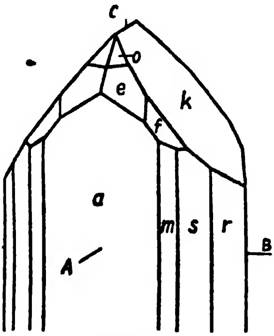


FIG. 21.

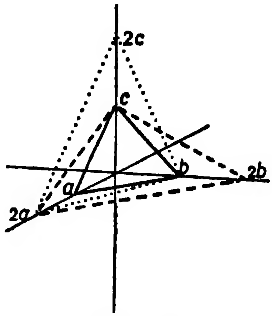


FIG. 22.

a : b : c. Then the intercepts of the eight faces tabulated on page 20 become :—

Face *a*,	a	:	∞b	:	∞c		Face *k*,	∞a	:	b	:	2c
„ *m*,	a	:	b	:	∞c		„ *e*,	a	:	b	:	c
„ *s*,	2a	:	b	:	∞c		„ *o*,	a	:	b	:	½c
„ *r*,	3a	:	b	:	∞c		„ *f*,	2a	:	b	:	2c

We may avoid the fraction in the intercepts of face *o* by writing them 2a : 2b : c, which are in the same ratio as a : b : ½c. The relation of faces *e*, *o* and *f* to the axes is shown in *Fig.* 22.

Parameters then are the units of measurement to be used on the different axes. At first sight it may seem unscientific to employ different units for measuring different directions of the same crystal; but in this we only follow Nature. By the use of parameters we are able to avoid long series of decimals and obtain instead simple whole numbers. The

law of rational intercepts, founded on the theories of Haüy and established by observation, states that any face of a crystal makes intercepts on the axes which may be expressed as rational multiples of the parameters. That is, the multiples are whole numbers, and not such quantities as $\sqrt{2}$, $\sqrt{3}$, or indeterminate decimals such as 0.75824. . . . In general they are quite low numbers, such as 1, 2 or 3, and rarely greater than 6, although of course infinity is quite common, indicating parallelism to an axis.

SYSTEMS OF CRYSTALS.

In the last chapter it appeared that all crystals belonged to one or other of thirty-two classes according to their symmetry, and that these classes fell into six or seven

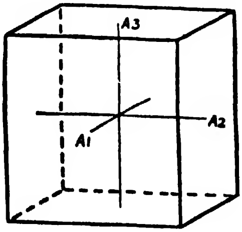


FIG. 23.
Cubic.

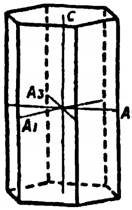


FIG. 24.
Hexagonal.

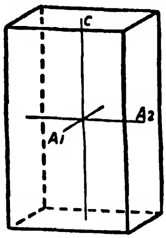


FIG. 25.
Tetragonal.

systems, which were based on the crystallographic axes. We are now able to define those systems.

1. *Cubic System.* Each of the three axes is at right angles to the other two, and the parameters are equal on all three axes.* *Fig.* 23 shows the cubic axes and a crystal form (the cube) in which each edge is parallel to one axis and each face is parallel to two axes.

2. *Hexagonal System.* In this system it is most convenient to take four axes; three of these are in one horizontal plane and make angles of 120° with one another, and the fourth is vertical and therefore perpendicular to the

* The statement that axes are equal or unequal should be avoided. Axes have direction only, and extend to infinity.

other three. The three horizontal (or lateral) axes have equal parameters; the vertical axis has a different parameter. See *Fig.* 24. The special axes sometimes employed for the trigonal system or sub-system will be explained later (p. 91).

3. *Tetragonal System.* The three axes are at right angles to one another. The two horizontal (or lateral) axes have equal parameters; that of the vertical axis is different. See *Fig.* 25.

4. *Orthorhombic System.* The three axes are at right angles to one another, and all three have different parameters. See *Fig.* 26.

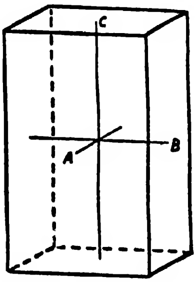


FIG. 26.
Orthorhombic.

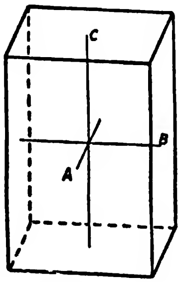


FIG. 27.
Monoclinic.

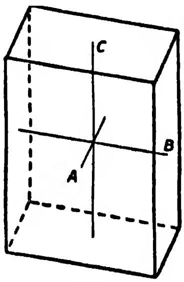


FIG. 28.
Triclinic.

5. *Monoclinic System.* One axis is at right angles to the other two, which are not at right angles to each other. The parameters are all different. See *Fig.* 27.

6. *Triclinic System.* None of the three axes is at right angles to another. The parameters are all different. See *Fig.* 28.

PRACTICAL WORK.

1. In topaz a : b : c=0.528542 : 1 : 0.476976. Express as multiples of the parameters the faces whose intercepts are 0.264271 : 1 : 0.238488; 1.057084 : 1 : 0.953952; and 0.528542 : ∞ : 0.158992.

2. Examine a number of crystal models, find the system to which each belongs and the position of its crystallographic axes.

CHAPTER IV.

INDICES.

In the last chapter it was shown that if the unit plane makes intercepts on the three axes proportional to a, b and c (the parameters), then any other face of the crystal will have intercepts that may be represented by pa, qb and rc, when p, q and r are whole numbers, usually small, or else infinity. We might therefore distinguish any face from the others by giving its intercepts, thus, a : b : ∞c, 2a : b : ∞c, or 2a : b : c. This is practically the rotation introduced by C. S. Weiss* in 1818.

A notation which, while not quite so simple at first sight as that of Weiss, leads to more useful results, is associated with the name of the late Prof. W. H. Miller, of Cambridge. He expressed a face by the numbers by which the parameters must be d i v i d e d to give the intercepts of that face. The intercepts of a face may be represented either by pa : qb : rc or by $\frac{a}{h} : \frac{b}{k} : \frac{c}{l}$. For instance, if 2a : 3b : 4c be the intercepts of a face, they may be divided by twelve without making any real difference, as they are only ratios, not absolute amounts. They will then become $\frac{a}{6} : \frac{b}{4} : \frac{c}{3}$. The numbers in the denominators are called the i n d i c e s. They are sufficient to identify the face. They are used without the parameters, thus, 643, which is, of course, read six four three, not six hundred and forty-three.

* There is an abbreviated form of the Weiss notation which was devised by C. F. Naumann, and another similar notation was employed by Dana, but as neither of these is now in use in this country, further reference to them is unnecessary.

As may easily be seen, the indices are equal to the parameters divided by the intercepts. Thus, a face with intercepts a : 2b : 2c would have indices $\frac{a}{a} : \frac{b}{2b} : \frac{c}{2c}$, or $1 : \frac{1}{2} : \frac{1}{2}$. As indices, like intercepts, are ratios, this may be written 211 to avoid fractions, and 211 is accordingly the Millerian symbol of a face whose intercepts are a : 2b : 2c.

Similarly a face with intercepts 3a : b : ∞c would have indices $\frac{a}{3a} : \frac{b}{b} : \frac{c}{\infty c}$, or $\frac{1}{3} : 1 : 0$, or 130, since any quantity divided by infinity is equal to zero. To take the general case, a face with intercepts pa : qb : rc will have indices $\frac{a}{pa} : \frac{b}{qb} : \frac{c}{rc}$, or $\frac{1}{p} : \frac{1}{q} : \frac{1}{r}$, or hkl say. Since p, q and r are rational, their reciprocals h, k and l must also be rational, and the law of rational intercepts may be restated as the law of rational indices in the following terms: If three axes are chosen parallel to three edges of a crystal which do not lie in one plane, and if the intercepts on these axes of the parametral plane are a : b : c, then the intercepts of any other face of the crystal can be expressed as $\frac{a}{h} : \frac{b}{k} : \frac{c}{l}$, where h, k and l are whole numbers or zero.

Since the indices are the reciprocals of the intercepts (measured in the respective parametral units), it follows that the larger the index the smaller is the corresponding intercept. The face 112 has half (not double) the intercept on the vertical axis that 111 has.

When the intercept on an axis is negative, the corresponding index is also negative. The face 111 occupies the front, right, top octant. The corresponding face in the front, left, top octant is $1\bar{1}1$. Those in the back right and left top octants are $\bar{1}11$ and $\bar{1}\bar{1}1$ respectively. The indices of the lower faces are similar except that the sign of the third index is negative.

The symbol of a face opposite and parallel to a given face is the symbol of the given face with all the signs changed. Thus the face opposite $1\bar{1}1$ is $\bar{1}1\bar{1}$.

Like the choice of axes, that of the parametral plane is

forms. A form which may be of great importance in one may be rare or show only small faces in another, or may not occur at all. The character of a crystal in respect of the presence and relative size of the faces of different forms constitutes its habit.

Not only is there marked individuality in the habit of crystals of different substances, but the same substance crystallising under changed conditions may show a considerable difference of habit.

In some cases, as explained in the first chapter, different conditions may cause an entirely different crystalline structure to be formed, the substance then being said to be dimorphic or polymorphic according as there are two or more different crystalline structures with the same chemical composition.

ISOMORPHS.

If the ultimate cells of two different substances resemble each other sufficiently in shape and size they may be able to combine in building up a joint crystal structure, either simultaneously or by successive accretions. An example of the former is the association of albite and anorthite, which form closely similar triclinic crystals and are found mixed in all proportions in the plagioclase series; while the latter may be illustrated experimentally by placing a crystal of common alum in a saturated solution of chrome alum (or *vice versa*). The original crystal will then be seen to continue its growth, but with different material, the presence of which will be indicated by the change of colour.

Two substances which crystallise in closely similar forms are said to be isomorphic. If their structures are so similar that they can form joint crystals in any proportion, as in the case of albite and anorthite, they are said to be perfectly isomorphic. If one can only crystallise with a limited amount of the other, as in the case of albite and orthoclase, they are described as imperfectly isomorphic.

ZONES.

In *Fig.* 30 it will be seen that the faces $10\bar{2}$, 100, 102 and 001 all meet in edges which are parallel to one another. They are, then, said to be in the same zone, and a line drawn through the intersection of the axes parallel to the edges in which they meet is called the zone-axis. Similarly the faces $2\bar{1}0$, 100, 210 and 010 all lie in another zone.

A zone-axis may be expressed by three indices included in a square bracket, thus [uvw], or in a particular case [234]. The latter symbol means that if a distance OU proportional to 2a be taken along OA, and then a distance UV proportional to 3b be taken parallel to OB, and finally a distance VW proportional to 4c be taken parallel to OC, then the line passing through O and W will be the zone-axis [234].

To obtain the symbol of the axis of the zone in which any two faces (not parallel to each other) lie, their indices hkl and h'k'l' are written down twice, one below the other, thus—

$$\begin{array}{cccccc} h & k & l & h & k & l \\ h' & k' & l' & h' & k' & l' \end{array}$$

The first and last pair of the indices are ignored, and the other four pairs are cross-multiplied as indicated by the arrows, descending lines being positive and ascending lines negative. This gives $(kl'-k'l)$, $(lh'-l'h)$, $(hk'-h'k)$ as the zone-symbol. The axis of the zone in which the faces 210 and 112 lie is $[2\bar{4}1]$, since cross-multiplication

$$\begin{array}{cccccc} 2 & 1 & 0 & 2 & 1 & 0 \\ 1 & 1 & 2 & 1 & 1 & 2 \end{array}$$

gives $[(1\times 2-1\times 0), (0\times 1-2\times 2), (2\times 1-1\times 1)] = [(2-0), (0-4), (2-1)] = [2\bar{4}1]$.

A zone-symbol, like the symbol of a face, is a ratio,

and may be multiplied or divided by any number, positive or negative, without altering its significance.

The indices of a face common to two zones whose zone-symbols are given may be found by cross-multiplying the zone-symbols in exactly the same way. In *Fig.* 30 the faces 100 and 102 lie in a zone whose symbol is [010], and the faces 100 and 210 in a zone whose symbol is [001]. The *face common to the two zones is found*, by *cross-multiplying* [010] and [001], to be either 100 or $\bar{1}00$ according to which zone-symbol is placed first. These are parallel faces, both *of which are common to the two zones.*

If a face hkl belongs to a zone [uvw], then $hu + kv + lw = 0$. Thus the face $10\bar{2}$ belongs to the zone $[2\bar{4}1]$ since $(1 \times 2) + (0 \times \bar{4}) + (\bar{2} \times 1) = 2 + 0 - 2 = 0$.

All planes which have rational indices, whether they have been observed as faces or not, are considered possible faces. In the same manner all lines which are parallel to zone axes with rational indices are considered to be possible edges.

PRACTICAL WORK.

1. Find the Millerian indices of the faces that have the following intercepts:—

a : b : 2c. ∞a : 3b : c. 3a : 2b : c.
a : b : ∞c. a : ∞b : ∞c. 2a : ∞b : c.

2. What are the intercepts of the following faces?

110 $20\bar{1}$ 010 001 432
120 $1\bar{2}0$ 123 643 12.12.1.

Sketch a set of axes and mark the position of these faces with reference to them.

3. Find the symbol of the zone in which the faces 102 and 122 lie.

4. Do the faces $1\bar{1}0$, 201 and 111 belong to the same zone?

CHAPTER V.

CRYSTAL MEASUREMENT AND REPRESENTATION.

The angles of crystals are measured by means of instruments known as g o n i o m e t e r s (Gr. *gonia*, angle, *metron*, measure). These are of two types, contact goniometers and reflecting goniometers.

Contact goniometers are somewhat crude instruments, useful for measuring the angles of wooden models and large crystals, especially those whose faces are too rough to give a good reflection. A convenient type, designed by Penfield, is shown in *Fig.* 31; it consists of a semicircular protractor in cardboard, to the centre of which a celluloid arm is pivoted. The angle between the arm and the base of the protractor is read by means of a fine line on the former, which passes through the centre of the semicircle.

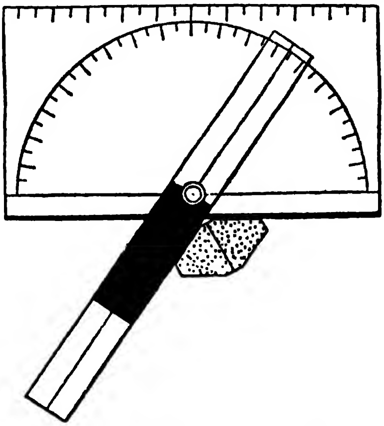


FIG. 31.—Contact Goniometer.

In using the contact goniometer the edge of the arm and the base of the protractor are brought into close contact with two faces of the crystal. The latter is held between the observer and the light, and the two parts or limbs of the goniometer are adjusted on the pivot until no light can be seen between their edges and the faces, with which they will then be in contact. Care must be taken that each limb is perpendicular to its respective crystal face.

The reflecting goniometer gives far more accurate results, especially on small crystals with brilliant reflecting

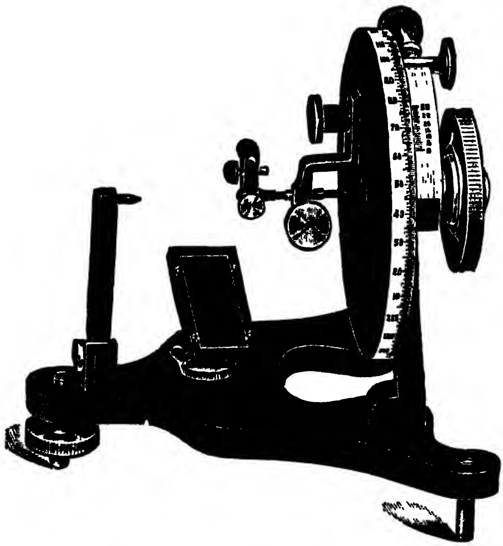


FIG. 32.—Reflecting Goniometer.

faces. The adjustment, however, of even the simplest type of reflecting goniometer is a somewhat elaborate process, and the complete reading of a crystal by its means requires much time and patience. A simple form of the instrument is shown in *Fig.* 32. This consists essentially of a graduated vertical circle which rotates, carrying with it the crystal-holder to which the crystal is attached by wax. There are adjusting screws for bringing the edge to be measured to the centre of the circle and in line with its

axis of rotation; and the circle is provided with a clamp, fine-adjustment, and vernier. A black glass mirror fixed on the base of the instrument, with its plane parallel to the axis of the circle, gives an image of a distant signal such as a horizontal window-bar, and an eye placed close to the crystal sees the same signal reflected in a crystal face. The two images coincide when the crystal face is parallel to the mirror. First one face and then the other is brought into this position, and the difference between the readings on the graduated circle gives the angle through which the crystal has been turned.

In *Fig.* 33, ABC represents in section the crystal edge to be measured, and DEF a ray of light from the distant signal, reflected from the face AB to the eye at F, where it coincides with the ray from the same signal reflected from the mirror M. The face BC will give a similar reflection of the signal when BC occupies the position of AB, that is, when the crystal has been turned through an angle HGK. The angle obtained from the reflecting goniometer therefore is not the solid angle ABC, but the angle between the normals to the faces AB and BC. This is evidently the supplement of ABC (*i.e.*, 180°—ABC), or the external angle that AB produced would make with BC. It is usual in crystallography to give this inter-normal or external angle in preference to the solid angle; thus the angle between two faces of a hexagonal prism (*Fig.* 24, p. 22) is given as 60°, not 120°.

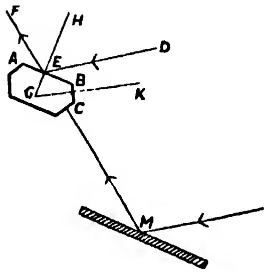


FIG. 33.—Principle of Reflecting Goniometer.

In the two-circle goniometers there are two axes of rotation and two graduated circles, and the readings give not the angles between the faces but the positions of the normals to the faces in polar co-ordinates, that is to say, by a system similar to that by which places on the earth's surface are determined by their longitude and latitude, except that the longitude or azimuth (ϕ) is measured all round in the same

direction from 0° to 360°, and, instead of the latitude, its complement, the polar distance (θ) is employed. By means of the formulæ of spherical trigonometry and the use of logarithmic tables it is possible to calculate from the polar co-ordinates of two faces the angle between them.

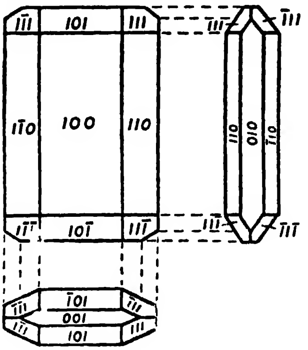
The angles between the crystallographic axes, and the parametral ratios, constitute what is known as the elements of the crystal, and from them it is possible to calculate the indices of any face whose angular position is known, or if the indices of a face be given to calculate its angular position. Inversely, from the angles between faces or from their positions as fixed by the two-circle goniometer, it is possible to determine the elements of the crystal and the indices of the faces.

PROJECTIONS.

There are several ways in which crystals may be represented on a plane surface, such as a sheet of paper. An ordinary perspective drawing, in which the actual appearance seen is projected on a vertical plane, has the objection that lines and planes which are really parallel appear to converge to some point behind the crystal. This "vanishing point" becomes more distant the farther the crystal is from the observer, and if the crystal is imagined to be at an infinite distance, so that all lines from it to the eye are parallel, then all parallel lines in the crystal will remain parallel in the drawing, or projection. The symmetry of the crystal is thus better indicated than in the ordinary perspective drawing. Where the line of sight is oblique to the plane of projection, the projection is said to be clinographic, and this projection is usually employed in text-books of crystallography or mineralogy. The point of view chosen is slightly above the plane of the axes OA and OB, and to the right of OA.

A clinographic projection of a crystal gives a fair idea of solidity; but to make an accurate projection of this sort an elaborate construction is required. For this reason a

simple form of o r t h o g r a p h i c projection, that is to say, a projection in which the plane of projection is at right angles to the line of sight and the object is supposed to be seen from an infinite distance, is to be preferred for elementary students. The most convenient method is to follow the example of architects and make three drawings of the crystal, front elevation, side elevation and ground plan. The first will represent the crystal as seen from an infinite distance directly in front, the second from the right, and the third from vertically above. The faces visible in each are marked with their indices and a table of the forms present, with their symbols, is added. *Fig.* 34 represents a



{100} Macropinakoid.
{010} Brachypinakoid.
{001} Basal pinakoid.
{110} Prism.
{101} Macrodome.
{111} Pyramid.

FIG. 34.—Olivine.

crystal of olivine treated in this way. The three views should be arranged with the side elevation on a level with and on the right of the front elevation, and with the view from above placed below the front elevation, so that the relation of corresponding points is clear. Broken lines connecting the same points in different views may be used to show this.

In crystals having rectangular axes the front view is taken from the positive direction of the axis OA, the side view from the positive direction of the axis OB, and the view from above from the positive direction of the axis OC. It follows that no face having a negative first index can appear in the front view, none with a negative second index

in the side view, and none with a negative third index in the view from above.

The front elevation may be regarded as a projection of the crystal on the plane of the axes OB and OC. If a crystal is placed on a sheet of paper, with these axes parallel to the paper, and perpendiculars to the paper are dropped from all points on the crystal, the feet of these perpendiculars form the front elevation of the crystal. A face perpendicular to the paper, 010 for example, will appear as a straight line

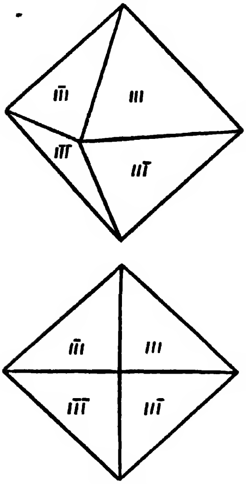


FIG. 35.
Octahedron.

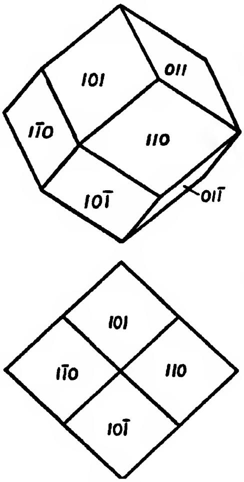


FIG. 36.
Rhombic Dodecahedron.

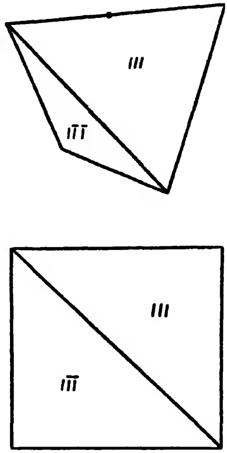


FIG. 37.
Tetrahedron.

in the projection, since the feet of all perpendiculars from the face will lie on a straight line. A face inclined to the plane of the paper, such as 110, will be more or less foreshortened, and only a face parallel to the paper, 100, will appear in its true proportions in the projection.

The front elevations of a few forms in the cubic system are given in *Figs.* 35 to 37, with their clinographic projections above them.

The stereographic projection, introduced

by Neumann in 1823, is the most useful projection for advanced work. It is less suitable for beginners, since it does not represent the appearance of the crystal but only the directions of its faces. The crystal is supposed to be placed at the centre of a sphere. Normals to the faces are drawn from the centre, and the point where each normal meets the surface of the sphere is known as the p o l e of the face through which it is drawn. Each face is thus represented by a point (its pole) on the surface of the sphere, B by B′, P by P′, and so on (*Fig.* 38). To project these points on to a plane, it is usual to choose a horizontal plane

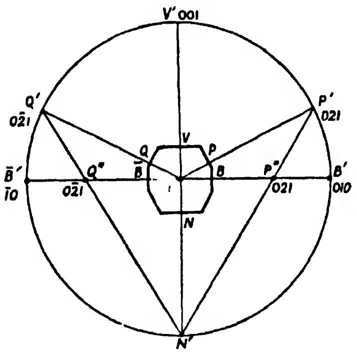


FIG. 38.—Stereographic Projection, Vertical Section.

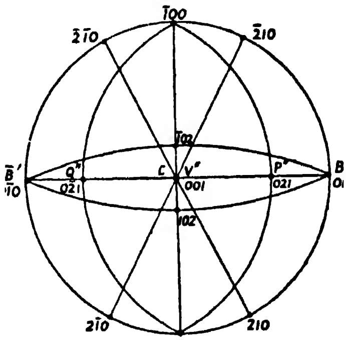


FIG. 39.—Stereographic Projection of *Fig.* 30.

of projection passing through the centre C of the sphere, and cutting the sphere in a horizontal circle known as the p r i m i t i v e c i r c l e. The eye is supposed to be placed on the surface of the sphere at the lower end of the vertical diameter at N′. The point where the line joining N′ to the pole of any face cuts the plane of projection is the projection of that pole. *Fig.* 39 shows a horizontal section through the sphere, and the paper is the horizontal plane of projection.

For example, P″ and Q″ are the projections of P′ and Q′, the poles of the faces P and Q, and B′ and $\bar{B}$′ are the poles and also the projections of the faces B and $\bar{B}$. If θ

be the angle between the direction CP of the normal to the face P and vertical CV, and r be the radius of the sphere, the distance CP″ from C the centre of the horizontal circle through $\bar{B}'B'$ will be equal to $r \tan \frac{\theta}{2}$.

It can be shown that all faces in the same zone (see p. 29) have their poles on the same great circle of the sphere, and every great circle is projected into a circle passing through the ends of a diameter of the primitive circle or into a diameter itself. Thus, in *Fig.* 39, the faces 100, 021 and $\bar{1}00$ are in one zone, and the faces 100, 102, 001, $\bar{1}02$ and $\bar{1}00$ are in another zone. It can also be shown that the angles between the projections of great circles are the same as those between the great circles themselves.

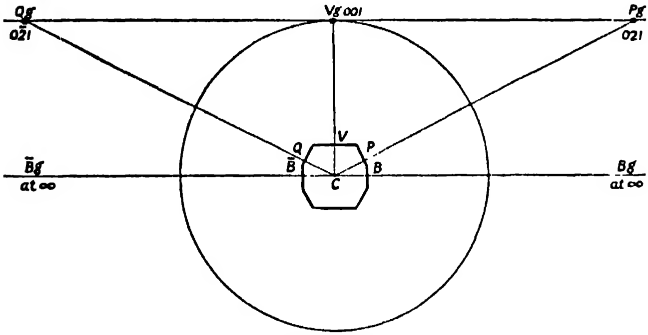


FIG. 40.—Gnomonic Projection; Vertical Section.

It is usually sufficient to show the poles in the upper hemisphere only. These are marked by dots on the projection. The poles in the lower hemisphere will appear outside the circumference of the primitive circle to an eye placed at N′.

This is the procedure for the purpose of the application of graphic methods of calculation, but in the case of faces, whose poles or projections on the sphere are near the point N′, the projections on the plane are at an inconvenient distance.

If it be desired to show within the circle the poles of faces below the crystal, this may be done in either of two ways:

(1) The under face may be indicated by the same point as the parallel upper face, that is to say the normal is continued through the centre till it meets the sphere on the opposite upper side, and the point where it emerges may be regarded as another pole of the face,

which may be projected on to the plane in the manner already described. If two opposite parallel faces are present which are equivalent, that is to say similar in all respects, they are indicated together by a simple dot; if there is only an upper face it is represented by +; if only an under face by a minus –.

(2) The other more usual but less scientific method is to transfer the eye to the upper end of the vertical diameter. The poles of the lower faces will then appear within the circle of the projection, and may be marked by small circles.

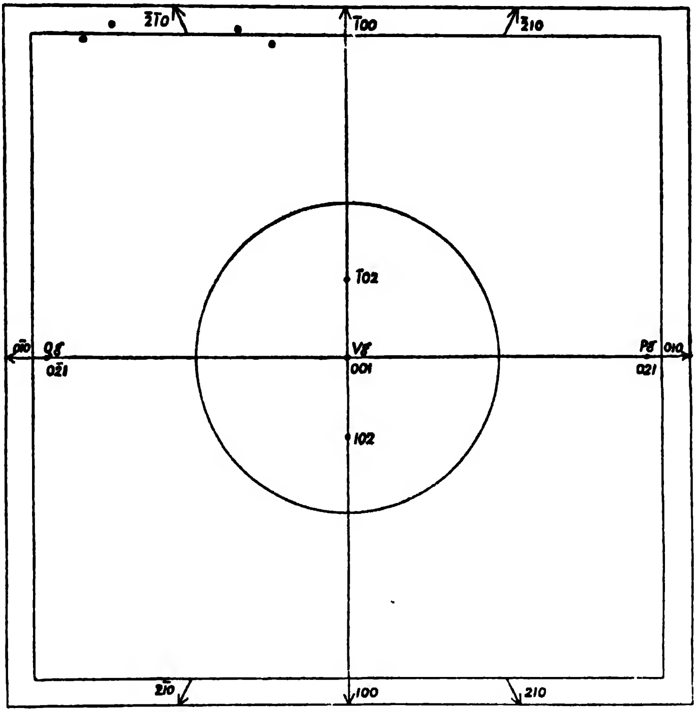


FIG. 41.—Gnomonic Projection of *Fig.* 30.

There is another kind of projection, known as the gnomonic projection, in which each face is represented by the point in which its normal meets a horizontal plane through v′, *Fig.* 38 (V_g, *Fig.* 40). Opposite faces are represented by the same point. This projection is illustrated by *Fig.* 40, which shows a section through the centre of the

crystal at right angles to the plane of projection, and by *Fig.* 41, which shows the plane of projection itself. In this projection the distance $v_g p_g$ of the projection p_g of a face p from the centre of projection will be $r \tan \theta$. In the gnomonic projection the points representing faces in the same zone are always in the same straight line.

By means of the stereographic or the gnomonic projection it is possible to make crystallographic calculations by graphic methods without the use of the formulæ of spherical trigonometry and logarithms. The results are not so exact, but are sufficiently accurate to afford a valuable check on the results obtained by the more elaborate means.

PRACTICAL WORK.

1. Measure the angles of crystal models with a contact goniometer. As a check, all the angles in a zone should be measured, when the sum of the inter-normal angles should be 360°. The sum of the solid angles should be $(n-2)180°$, where n is the number of faces in the zone.

2. Practise the measurement of crystals with some form of reflecting goniometer, and compare the readings with the angles given in Dana's "System of Mineralogy."

CHAPTER VI.

ORTHORHOMBIC SYSTEM—OLIVINE CLASS
(IIDc, ORTHORHOMBIC CENTRAL).*

In the orthorhombic system the axes are three in number; they are all at right angles to one another, and have different parameters. The ratio of the parameters is different in different minerals but constant in the same mineral, and the necessary e l e m e n t s from which the inclination of any face of a mineral can be calculated are the two ratios a : b and b : c. The value of b is always taken as unity, and the elements of an orthorhombic mineral are succinctly stated as a : b : c = 0.46575 : 1 : 0.58651 (in the case of olivine).

The ratio a : b is always less than unity; that is, the parametral plane makes a smaller intercept on the front and rear axis OA (*Fig.* 42) than on the right and left axis OB. The former is therefore called the b r a c h y - a x i s (Gr. *brachys*, short) and the latter the m a c r o - a x i s (Gr. *makros*, long). The intercept of the parametral plane on the vertical axis OC may be greater or less than the intercept on the other axes. The initials of brachy-axis, macro-axis and vertical axis, OA, OB and OC respectively, are in alphabetical order and serve as a mnemonic.†

In the olivine class the crystallographic axes are axes of half-turn symmetry; they lie in three planes of symmetry

* This system and class are taken first because they are more readily understood by the beginner than the others. The symbols and alternative titles of the classes are explained in Chapter XV.

† To avoid any ambiguity as to the position of the axes, some writers make use of the short and long signs, ˘ and ¯, for the brachy- and macro-axes, and ′ for the vertical axis, thus, $\breve{a}$, $\bar{b}$, $\acute{c}$.

(*Fig.* 43), and intersect in a centre of symmetry. The three planes of symmetry intersect in the three crystallographic axes and partition the space surrounding the crystal into eight separate compartments or octants. From this symmetry may be deduced the number of faces present in each form.

The parametral plane 111 meets all three axes in their positive directions, and occupies the front, right, top octant, which may be described as the positive octant. Since the vertical plane AOC is a plane of symmetry, there must be a corresponding face $1\bar{1}1$ in the front, left, top octant. The vertical plane BOC is also a plane of

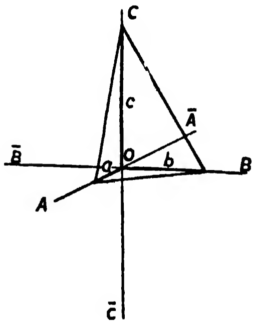


FIG. 42. — Orthorhombic Axes and Parametral Plane.

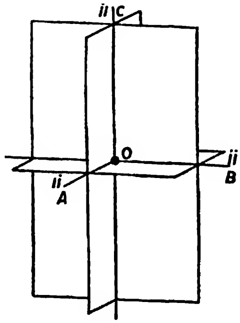


FIG. 43.—Olivine Class, Symmetry, C, 3P, 3 ii.

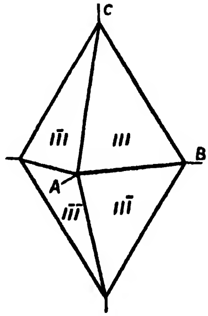


FIG. 44.—Pyramid.

symmetry; accordingly these two faces must be accompanied by two others, $\bar{1}11$ and $\bar{1}\bar{1}1$, in the back, top octants; and similarly the horizontal plane of symmetry AOB necessitates four corresponding faces, $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$ and $1\bar{1}\bar{1}$ in the four lower octants. The lines and centre of symmetry do not give rise to any additional faces, and therefore the complete form {111} consists of the eight faces following:—

$$111,\ \bar{1}11,\ \bar{1}\bar{1}1,\ 1\bar{1}1,$$
$$11\bar{1},\ \bar{1}1\bar{1},\ \bar{1}\bar{1}\bar{1},\ 1\bar{1}\bar{1},$$

the indices of which show all possible variations in sign. This form (*Fig.* 44) is known as a pyramid. When it exists alone the faces are triangles, and the form differs from

the octahedron of the cubic system in that the faces are not equilateral, but scalene triangles.

A pyramid in crystallography resembles two of the solid bodies generally known as pyramids placed base to base. The term bipyramid is therefore employed for this form by some writers.

The form {111} is called the unit pyramid to distinguish it from other pyramids, such as {112}, {123}, {331}. So long as each face cuts all three axes at finite distances, consideration of the symmetry of the class shows that the form will consist of eight faces, each having the same indices in the same order, but with all possible changes of sign. Thus the general form {hkl} consists of the faces

$$h k l,\ \bar{h} k l,\ \bar{h} \bar{k} l,\ h \bar{k} l,$$
$$h k \bar{l},\ \bar{h} k \bar{l},\ \bar{h} \bar{k} \bar{l},\ h \bar{k} \bar{l}.$$

Such faces as khl and lhk are not included in the form {hkl}, for they are not similar in position or in physical characters.

It is evident that the form {113} is a flatter pyramid, and {331} or {11⅓} a steeper pyramid than {111}. The greater the first two indices, as compared with the third, the steeper will be the pyramid. The limit is reached when the faces become parallel to the vertical axis. Thus the pyramid $\{11\frac{1}{n}\}$ becomes steeper as n increases, and when n is infinite and the faces are parallel to the vertical axis, the symbol of the form becomes {110}. If the intercepts on

* The faces meet the axial planes in rhombs, that is to say, four-sided figures whose sides are equal but whose angles are not right angles. Hence the name orthorhombic. The prefix refers to the fact that the vertical axis is at right angles to the rhomb which has the macro- and brachy-axes as its diagonals. Hence these axes are sometimes termed the macro- and brachy-diagonals. It is incorrect to speak of the rhombic system.

Formerly the lateral axes were taken parallel to the sides of the rhomb, and had equal parameters, but were not at right angles to each other. Such axes present advantages where there are strong prismatic cleavages, as they probably correspond to the primitive cell, but they are never employed now. See p. 52, note.

the A and B axes are not in the parametral ratio, the symbol has the general form {hko}.

The face 110 must be accompanied by a corresponding face $1\bar{1}0$ to the left of the symmetry-plane passing through the axes OA and OC, and by the faces $\bar{1}10$ and $\bar{1}\bar{1}0$ at the back of the symmetry-plane passing through the axes OB and OC. The horizontal plane of symmetry gives rise to no additional face, since there is no difference between 110 and $11\bar{0}$, for zero has no sign, and the mirror-image of a plane perpendicular to the mirror is a continuation of the plane.

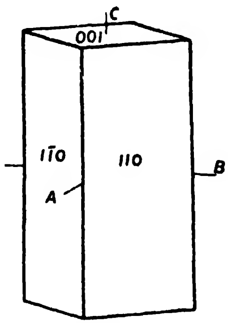


FIG. 45.—Prism.

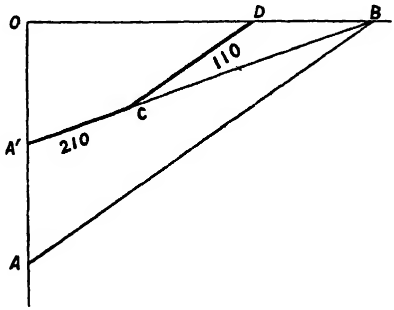


FIG. 46.—Two Prisms, part projection on horizontal plane.

Accordingly the form {110} consists of four faces only (*Fig.* 45), and the same is true of the forms {120}, {310}, and in general terms {hko}. These forms, which are parallel to the vertical axis and cut the other two, are called p r i s m s .

A prism is an open form and does not entirely enclose a space, as a pyramid does. It cannot, therefore, exist alone, but must be terminated by some other form, such as a pyramid. In *Fig.* 45 it is terminated by the form {001}.

If two prisms, such as {110} and {210}, exist together in the same crystal, they may be distinguished from each other by considering their intercepts on the A and B axes. *Fig.* 46 represents the positive directions of these axes as seen from above, and OA : OB the ratio of their parameters a : b. Then AB is the trace of the face 110.

The face 210 will have intercepts ½a and b on the A and B axes respectively; it will therefore be represented by A′B, where OA′ is half OA. The two faces will not both appear if one is wholly within the other, as A′B is within AB; but if from any point C on A′B we draw CD parallel to AB, CD will still represent the trace of 110, and A′C and CD therefore represent 210 and 110 respectively. Note that 210, with a bigger first index than 110, lies nearer the first axis, OA. The faces 120 would lie nearer the second axis than 110. The general rule, that the greater the index, the nearer the face is to the corresponding axis, is worth remembering.

A form with faces parallel to one of the horizontal axes

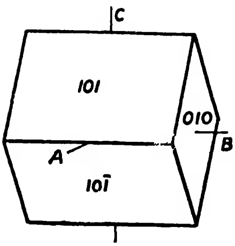


FIG. 47. Macrodome.

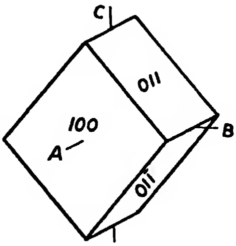


FIG 48. Brachydome.

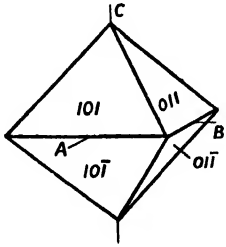


FIG. 49.—Macrodome and Brachydome.

and cutting the other two axes, like a horizontal prism, is, rather inappropriately, known as a d o m e , because its two upper faces bear some resemblance to the roof of a house (L. *domus*). The two domes are distinguished from each other by the same prefix as the axis to which they are parallel. The m a c r o d o m e is parallel to the macro-axis and cuts the axes of A and C; the symbol of its general form is {hol}. The b r a c h y d o m e is parallel to the brachy-axis and cuts the axes of B and C; its general form is {okl}. As in the case of the prism, it will be seen that each dome consists of four faces and that it requires some other form to complete the crystal. *Fig.* 47 is a combination of a macrodome {101} with the form {010}; *Fig.* 48 a brachydome {011} with {100}; and *Fig.* 49 a combina-

tion of macrodome and brachydome. This combination differs from the pyramid in that the horizontal edges form a rectangle, whereas those of the orthorhombic pyramid form a rhomb. Again, the planes of symmetry bisect the faces of a combination of the two domes; but they pass through the edges of a pyramid.

The general form of a prism may be written $\{h10\}$ or $\{1\frac{1}{h}0\}$. If h is very large the two front faces will be nearly at right angles to the brachy-axis and make a very obtuse angle with each other, and the same will be the case with the two back faces. If h becomes infinite they will coincide, and the form will become $\{100\}$, which will consist of two

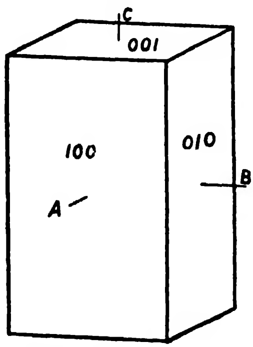


FIG. 50.—Pinakoids.

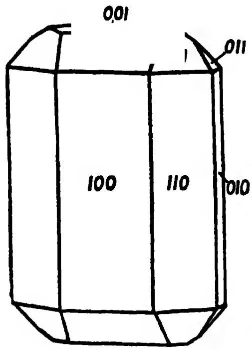


FIG. 51.—Olivine.

faces, a front face 100 and a back face $\bar{1}00$, as required by the plane of symmetry COB, the digonal axes OB and OC, and the centre of symmetry.

In like manner, the form $\{010\}$ consists of the two faces 010 and $0\bar{1}0$; and $\{001\}$ consists of the two faces 001 and $00\bar{1}$. These forms, which are parallel to two of the crystallographic axes, and therefore consist of two faces only, are called p i n a k o i d s* (Gr. *pinax*, a plank, *eidos*, form).

* Some writers regard any form of two parallel faces as a pinakoid, but it is usually restricted to those which are parallel to two axes, as is always the case in the olivine class.

Fig. 50 shows a combination of the three pinakoids. The one with faces in front and behind, {100}, is called the macropinakoid; it is parallel to the macro-axis and the vertical axis. The one at the sides, {010}, parallel to the brachy-axis and the vertical axis, is called the brachypinakoid. The one at the top and bottom, {001}, parallel to both the horizontal axes, is known as the basal pinakoid.

Summarising, the forms occurring in the olivine class of the orthorhombic system may be tabulated as follows:—

Name of form.	General symbol.	Unit form.	Number of faces.
Pyramid	{hkl}	{111}	8
Prism	{hko}	{110}	4
Macrodome	{hol}	{101}	4
Brachydome	{okl}	{011}	4
Macropinakoid	{100}		2
Brachypinakoid	{010}		2
Basal pinakoid	{001}		2

PRACTICAL WORK.

Make drawings of models showing the simple forms of the orthorhombic system—pyramid, prism, domes and pinakoids. Mark each face shown in the drawings with its symbol, and name the form or forms present, as shown on page 35.

Crystals or models of the more important orthorhombic minerals should also be drawn. Some of these are named below, with their elements and the forms commonly present.

Olivine. a : b : c=0.46575 : 1 : 0.58651. Some crystals have {100} well developed, with {110}, {010}, {001}, {101} and {111} (*Figs.* 34, 51). Others have a steep brachydome, {021} as a dominant form, with or without {100} (*Fig.* 21). Other forms commonly present include {120}, {121} and {011}.

Topaz. a : b : c=0.52854 : 1 : 0.47698. Two prisms frequently occur, {110} and {120}, sometimes with {010}.

These are terminated by numerous small faces, belonging to forms such as {021}, {111}, {223}, {221} and {001}. Most specimens show at one end the perfect basal cleavage, and so have a uniterminal appearance (*Fig.* 52).

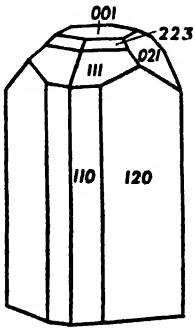


FIG. 52.—Topaz.

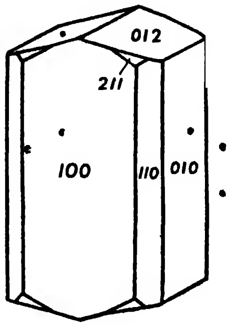


FIG. 53.—Hypersthene.

Hypersthene. a : b : c=0.97133 : 1 : 0.57037. The pinakoids {100} and {010} have their vertical edges modified by the prism {110}, which makes angles of about 44° and 46° with them. The terminal forms may include {111}, {212}, {232}, etc. (*Fig.* 53).

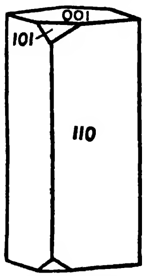


FIG. 54.—Andalusite.

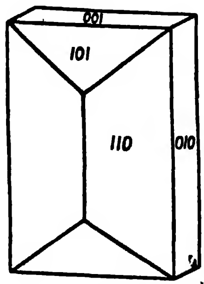


FIG. 55.—Staurolite.

Andalusite. a : b : c=0.98613 : 1 : 0.70245. The prism {110} has an angle of 89° 12′, and is terminated by {001}, sometimes with {011} or {101} (*Fig.* 54).

Sillimanite. a : b=0.970 : 1. The unit prism has an angle of 88° 15′, and may be accompanied by {230} with an

angle of 69°. The crystals are long and slender, and no measurable terminal faces occur, so that the element c is not known.

Staurolite. a : b : c=0.4734 : 1 : 0.6828. The prism angle is only 50° 40′, owing to the low ratio of a : b. The forms {010} and {001} are usually present with {110}, and sometimes {101} also (*Fig.* 55).

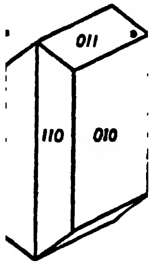


FIG. 56.—Aragonite.

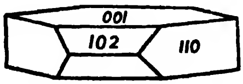


FIG. 57.—Barytes.

Aragonite. a : b : c=0.62244 : 1 : 0.72056. Simple crystals (*Fig.* 56) may show {110}, {010} and {011}, but this mineral is usually twinned.

Barytes. a : b : c=0.81520 : 1 : 1.31359. This mineral exhibits a variety of habits, but is usually elongated on the A or B axis. A common combination is shown in *Fig.* 57, with {001}, {110} and {102}.

Brookite. a : b : c=0.84158 : 1 : 0.94439. The habit is usually tabular, with {100} dominant, and showing {210}, {110} and {010} at the sides and {001}, {102}, {122} and {021} at the ends.

CHAPTER VII.

MONOCLINIC SYSTEM—AUGITE CLASS.

(IIMc, MONOCLINIC CENTRAL).

In the monoclinic system two of the axes are not at right angles to each other, but the third is at right angles to both. All three axes have different parameters. The elements of a monoclinic mineral are therefore a : b : c and β, the acute angle between the axes of A and C, which are not at right angles. The ratios a : b and b : c may be greater or less than unity.

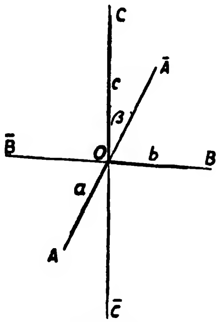


FIG. 58.—Monoclinic Axes.

A monoclinic crystal is held so that the axis which is perpendicular to both the others is right and left (*Fig.* 58). It is known as the o r t h o - a x i s (Gr. *orthos*, straight, upright).* The other two axes then lie in a vertical front-to-

* It would have been better if the ortho-axis, which is essentially different from the other two, had been placed in the vertical position; the other axes would then have been macro- and brachy-axes as in the orthorhombic and triclinic systems; but it is too late to make a change.

back plane. They are placed so that one of them is vertical, and the other slopes down toward the observer. This is called the c l i n o - a x i s (Gr. *klinein*, to incline).

Thus OA is the clino-axis,
OB ,, ,, ortho-axis,
and OC ,, ,, vertical axis,

and here again the alphabetical order of the axes is an aid to the memory.

In the augite class there is a centre of symmetry (see *Fig.* 59), one axis of half-turn symmetry, the ortho-axis, and one plane of symmetry in which the two other axes lie.

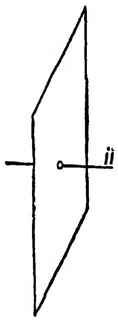


FIG. 59.—Augite Class, Symmetry, C, P, ii.

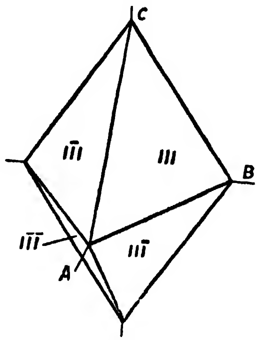


FIG. 60.—Hemi-pyramids.

The parametral face 111 must be accompanied by the corresponding face $1\bar{1}1$ on the other side of the plane of symmetry AOC. The centre of symmetry requires a face $\bar{1}\bar{1}\bar{1}$ opposite 111 and $\bar{1}1\bar{1}$ opposite $1\bar{1}1$. The line of symmetry OB also gives the same two faces. Therefore the form $\{111\}$ consists of four faces, not eight as in the orthorhombic system. This is because there is only one plane of symmetry.

The face $\bar{1}11$ must similarly be accompanied by three other faces, $\bar{1}\bar{1}1$, $1\bar{1}\bar{1}$ and $11\bar{1}$. There are therefore two entirely independent forms, $\{111\}$ and $\{\bar{1}11\}$, each consist-

* The signs \`, ¯ and ' are sometimes useful to distinguish the clino-, ortho- and vertical axes, thus à, b̄, c̍.

ing of four faces and known as a hemi-pyramid. *Fig.* 60 shows these two forms in combination, but it must be clearly understood that either may be present in a crystal without the other.* A single hemi-pyramid resembles a dome in the orthorhombic system, but each face cuts three axes, while a dome-face cuts only two. A second form of some kind is necessary to complete the crystal, for a hemi-pyramid is not a closed form. There are indeed no closed forms in this system, and a monoclinic crystal is always a combination of forms.

Similar reasoning shows that other forms cutting all three axes, such as $\{112\}$, $\{\bar{3}21\}$, and in general $\{hkl\}$ and $\{\bar{h}kl\}$, are hemi-pyramids with four faces each. It will be noticed that all the faces comprised in the form $\{hkl\}$ occur in the four octants including the obtuse axial angles AOC and $\bar{A}O\bar{C}$, while the faces in $\{\bar{h}kl\}$ subtend the acute angles $CO\bar{A}$ and $AO\bar{C}$. The two hemi-pyramids are distinguished as positive and negative, $\{hkl\}$ being the negative and $\{\bar{h}kl\}$ the positive hemi-pyramid. The choice is inexplicable and unfortunate, since the negative hemi-pyramid includes the face with all positive indices, hkl, and its faces are in general larger than those of the positive form. It may be noted too that in the negative forms the product of the first and third indices is always positive, and in the positive forms it is always negative. The only consideration that connects a negative sign with the obtuse octants is that the tangent and the cosine of an obtuse angle are negative. It would have

* A combination of two corresponding hemi-pyramids meets the plane AOB in a rhomb, but the vertical axis is inclined to it. Hence this system was formerly termed the clinorhombic, as opposed to the orthorhombic. The ortho- and clino-axes are parallel to the diagonals of the rhomb, and are often referred to as the ortho- and clino-diagonals. The lateral axes were formerly taken parallel to the sides of the rhomb. They had equal parameters, but were not at right angles to each other or to the vertical axis, with which they made equal angles. As in the orthorhombic system, these rhombic axes have advantages where there are strong prismatic cleavages; but they are not now employed. See p. 43, note.

been better to use the terms obtuse and acute hemi-pyramids, but here again it seems too late now to change.

The following are the faces of the hemi-pyramids in general terms:—

Positive (or acute) hemi-pyramid.	Negative (or obtuse). hemi-pyramid.
$\bar{h}kl$, $\bar{h}\bar{k}l$,	hkl, $h\bar{k}l$,
$h\bar{k}\bar{l}$, $hk\bar{l}$.	$\bar{h}\bar{k}\bar{l}$, $\bar{h}k\bar{l}$.

The face 110 must be accompanied by $1\bar{1}0$, since AOC (*Fig.* 58) is a plane of symmetry, by $\bar{1}10$ since $B\bar{B}$ is a line of symmetry, and by $\bar{1}\bar{1}0$ since O is a centre of symmetry. The form {110} is therefore a p r i s m of four faces (*Fig.* 61), as

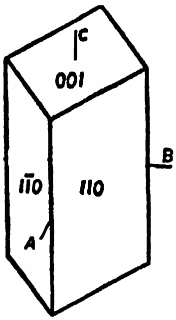


Fig. 61.—Prism.

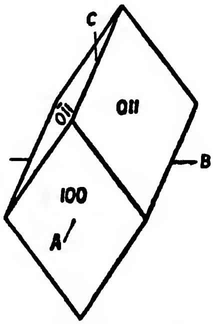


Fig. 62.—Clinodome.

in the orthorhombic system. Any other form parallel to the vertical axis and cutting the other two, such as {210} or {320}, is also a prism, and in general terms the prism {hko} consists of the four faces, hko, $\bar{h}ko$, $\bar{h}\bar{k}o$, and $h\bar{k}o$.

The face 011 must be accompanied by $0\bar{1}1$, on the opposite side of the plane of symmetry, and the centre and line of symmetry give two more faces, $0\bar{1}\bar{1}$ and $01\bar{1}$. The form {011} consists therefore of four faces (*Fig.* 62); it is a dome, and, since it is parallel to the clino-axis, it is called a c l i n o d o m e. The general form of the clinodome, {okl}, consists of the four faces—

$$okl,\ o\bar{k}l,$$
$$o\bar{k}\bar{l},\ ok\bar{l}.$$

The face 101 is perpendicular to the plane of symmetry, which is therefore inoperative, but either the centre or the line of symmetry will necessitate the opposite face $\bar{1}0\bar{1}$. The form {101} therefore consists of two faces,* and since they are parallel to the ortho-axis and not to either of the other axes, it is termed a h e m i - o r t h o d o m e . Another and independent hemi-orthodome consists of the pair of faces $\bar{1}01$ and $10\bar{1}$. The former, opposite the obtuse axial angle, is the negative or obtuse hemi-orthodome, the latter the positive or acute hemi-orthodome. *Fig.* 63 shows a

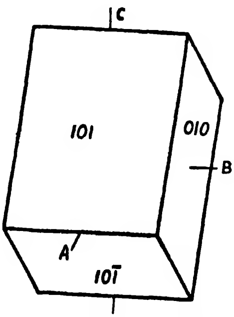


FIG. 63.—Hemi-orthodomes.

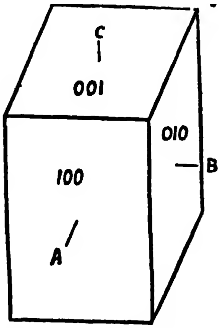


FIG. 64.—Pinakoids.

combination of two hemi-orthodomes and {010}. In general terms these two forms consist of the following faces :

Positive (or acute) hemi-orthodome.	Negative (or obtuse) hemi-orthodome.
$\bar{h}0l$,	$h0l$,
$h0\bar{l}$.	$\bar{h}0\bar{l}$.

Here also the product of the first and third indices is positive in the negative form and negative in the positive form.

Each of the forms parallel to two axes consists of two faces only, as in the orthorhombic system. They are distinguished as the orthopinakoid, {100}, parallel to the ortho- and vertical axes; the clinopinakoid, {010}, parallel to the

* It would, therefore, be called a pinakoid by some writers.

clino- and vertical axes; and the basal pinakoid, {001}, parallel to the ortho- and clino-axes.

Fig. 64 shows a combination of the three pinakoids. Their component faces are—

Orthopinakoid 100, $\bar{1}00$,
Clinopinakoid 010, $0\bar{1}0$,
Basal pinakoid 001, $00\bar{1}$.

The following table is a summary of the forms occurring in the orthoclase class of the monoclinic system.

Name of form.	General symbol.	Unit form.	Number of faces.
Positive hemi-pyramid	$\{\bar{h}kl\}$	$\{\bar{1}11\}$	4
Negative hemi-pyramid	{hkl}	{111}	4
Prism	{hko}	{110}	4
Clinodome	{okl}	{011}	4
Positive hemi-orthodome	$\{\bar{h}ol\}$	$\{\bar{1}01\}$	2
Negative hemi-orthodome	{hol}	{101}	2
Orthopinakoid	{100}		2
Clinopinakoid	{010}		2
Basal pinakoid	{001}		2

It should be noticed that all the forms consisting of four faces are oblique to the plane of symmetry. They are all essentially similar, and the distinction drawn between them is purely conventional. The forms consisting of two faces fall into two classes—the clinopinakoid, which is parallel to the plane of symmetry, and the remainder which are at right angles to it. These latter again are all essentially similar.

PRACTICAL WORK.

Orthoclase. a : b : c=0.65851 : 1 : 0.55538; $\beta=63^\circ\ 56'\ 46''$. Commonly tabular parallel to 010, or prismatic with the prism {110} and clinopinakoid {010} well developed, terminated by the basal pinakoid {001} and one or both of the positive hemi-orthodomes $\{\bar{1}01\}$ and $\{\bar{2}01\}$. The edges may be modified by a clinodome {011} or a posi-

tive hemi-pyramid {$\bar{1}11$} as in *Fig.* 65. In adularia {010} is absent or small. In the Baveno habit (*Fig.* 66) the elongation is along the axis of A, not C, and the pinakoids {010}

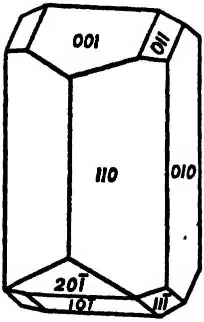


FIG. 65.—Orthoclase.

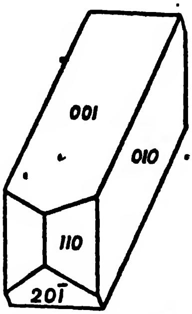


FIG. 66.—Orthoclase.

and {001} are dominant with a short prism {110} and usually {$\bar{2}01$}.

Augite. a : b : c = 1.0921 : 1 : 0.5893; β = 74° 10′. The orthopinakoid {100} is usually well developed, with the prism {110} and clinopinakoid {010}, and terminated by a

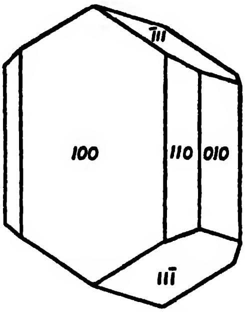


FIG. 67.—Augite.

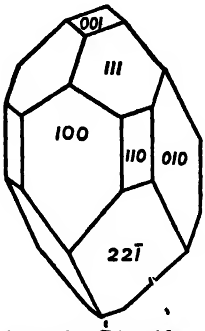


FIG. 68.—Diopside.

positive hemi-pyramid {$\bar{1}11$}, as in *Fig.* 67. In *diopside* (*Fig.* 68), another monoclinic pyroxene, the basal pinakoid is often present and the pinakoids {100} and {010} are more equally developed.

It should be noticed that in the pyroxenes the ratio a : b

is very near unity (compare *hypersthene*, an orthorhombic pyroxene, p. 48). This gives a prism angle of about 90°, by which they may be distinguished from minerals of the amphibole group, in which the prism angle is not quite 60°.

Hornblende. a : b : c=0.55108 : 1 : 0.29376; $\beta=73°$ 58′. In the amphiboles, of which hornblende is an example, a prismatic habit is usual, formed by the prism {110} and the clinopinakoid {010}. The orthopinakoid {100} is rarely seen, and a cross-section is therefore six-sided, not eight-sided as in the pyroxenes. The prism angle is about 56°. The commonest terminations are the clinodome {011} and the positive hemi-orthodome {$\bar{1}$01}, as in *Fig.* 69.

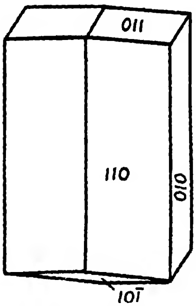


FIG. 69.—Hornblende.

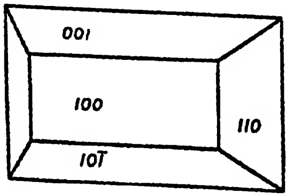


FIG. 70.—Epidote.

Epidote. a : b : c=1.5787 : 1 : 1.8036; $\beta=64°$ 37′. Elongation on the B axis is usual, crystals often showing orthopinakoid {100}, basal pinakoid {001} and positive hemi-orthodome {$\bar{1}$01} or {$\bar{2}$01}, perhaps terminated by short prism faces at one end only (*Fig.* 70).

Muscovite. a : b : c=0.57735 : 1 : 3.3128; $\beta=89°$ 54′. The micas exhibit pseudo-hexagonal crystals, formed by the prism {110} and the clinopinakoid {010}, terminated by the basal pinakoid {001}, which is the cleavage direction.

Titanite (Sphene). a : b : c=0.75467 : 1 : 0.85429; $\beta=60°$ 17′. Among a variety of types, that shown in *Fig.* 71 is one of the simplest. The negative hemi-pyramid {111} and the orthopinakoid {100} are dominant forms, the

prism {110} is poorly developed, and the basal pinakoid {001} and negative hemi-orthodome {102} are also present.

Gypsum. a : b : c=0.68994 : 1 : 0.41241 ; $\beta=80^\circ 42'$. Crystals of gypsum (selenite) usually show the clinopinakoid {010}, prism {110} with an angle of $68^\circ 30'$, and negative

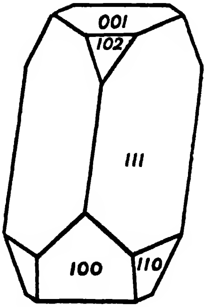


FIG. 71.—Titanite.

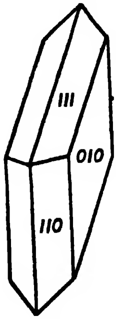


FIG 72.—Gypsum

hemi-pyramid {111} with a more obtuse edge, $36^\circ 12'$. This edge is inclined at $52^\circ 25'$ to the vertical; and as β is $80^\circ 42'$, it is evident that the faces meeting in it will cut the A axis as well as B and C, and are faces of a hemi-pyramid, not a clinodome as might be supposed, and as it would have been if the clino-axis had been chosen parallel to this edge (*Fig.* 72).

In the monoclinic system the clino- and vertical axes may be taken parallel to any edges in the plane of symmetry, and different authors do not always agree as to the choice of these axes. The ortho-axis, on the other hand, must always coincide with the line of symmetry.

CHAPTER VIII.

TRICLINIC SYSTEM—ALBITE CLASS
(IMc, TRICLINIC CENTRAL).

In the triclinic system none of the three axes is at right angles to another, and all have different parameters. The elements of a triclinic mineral are therefore the parametral ratio a : b : c and the three angles, α, β and γ, where γ is the angle between the positive directions of the axes of B and C, β the angle between the positive directions of the axes of C and A, and α the angle between the positive directions of the axes of A and B, as shown in *Fig.* 73.

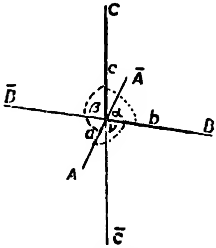


FIG. 73.—Triclinic Axes.

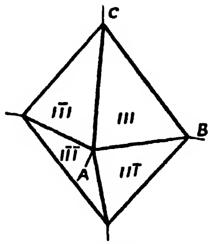


FIG. 74.—Quarter-Pyramids.

The choice of the axial directions is quite conventional, and reference must be made to published descriptions in the case of each mineral. The crystal is held with the axis of C vertical, the axis of A sloping downward from back to front, and the axis of B usually approximately from left to right and inclined downward in that direction. Usually the parameter b is greater than a. The axes are named as in the orthorhombic system, the axis of A being the brachy-axis, that of B the macro-axis, and that of C the vertical axis.

In the albite class, to which all known triclinic minerals belong, there is a centre of symmetry but no planes or lines of symmetry. The effect of a centre of symmetry is to duplicate any face by a similar parallel face in the opposite octant. Every form in this class therefore consists of a pair of similar opposite parallel faces, and agrees with the definition of a pinakoid as the term is employed by some crystallographers. It is convenient, however, to restrict it to forms parallel to two axes, and not to extend it to those meeting two or all three axes.

A form cutting all three axes is called a quarter-pyramid, because it has only a quarter of the eight faces in

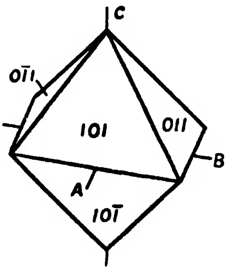


Fig. 75.—Hemi-macrodomes and Hemi-brachydomes.

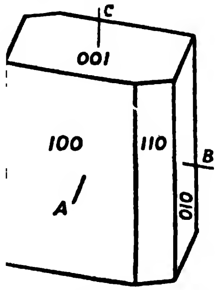


Fig. 76.—Pinakoids and Hemi-prism.

a complete pyramid. Thus the form {111} includes the two opposite faces 111 and $\bar{1}\bar{1}\bar{1}$ only. *Fig.* 74 shows four quarter-pyramids in combination; their faces will differ in size, shape, and physical characters, the largest being generally opposite the obtuse axial angles.

Forms parallel to one axis and cutting the other two are known as hemi-brachydomes, hemi-macrodomes, or hemi-prisms, according as they are parallel to the brachy-axis, macro-axis, or vertical axis.

Forms parallel to two axes are the macropinakoid {100}, brachypinakoid {010}, and basal pinakoid {001}.

Fig. 75 shows two hemi-macrodomes and two hemi-brachydomes, while *Fig.* 76 shows the three pinakoids in combination with a hemi-prism.

The forms occurring in the albite class of the triclinic system are therefore as follows :—

Name of form.	General symbol.	Unit form.	Number of faces.
Four quarter-pyramids	$\{hk\bar{l}\}$, etc.	$\{111\}$, etc.	2
Two hemi-brachydomes	$\{okl\}$, $\{o\bar{k}l\}$	$\{011\}$, $\{0\bar{1}1\}$	2
Two hemi-macrodomes	$\{hol\}$, $\{\bar{h}ol\}$	$\{101\}$, $\{\bar{1}01\}$	2
Two hemi-prisms	$\{hko\}$, $\{h\bar{k}o\}$	$\{110\}$, $\{1\bar{1}0\}$	2
Macropinakoid	$\{100\}$		2
Brachypinakoid	$\{010\}$		2
Basal pinakoid	$\{001\}$		2

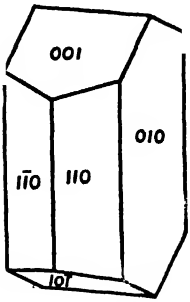


FIG. 77.
Plagioclase.

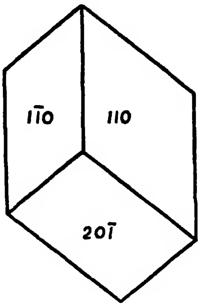


FIG. 78.
Plagioclase.

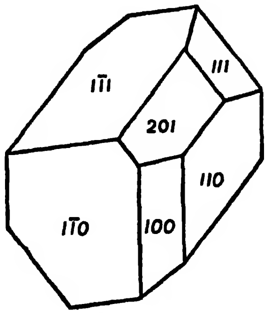


FIG. 79
Axinite.

PRACTICAL WORK.

Plagioclase (Albite). a : b : c=0.6335 : 1 : 0.5577; $\alpha=94°\ 3'$, $\beta=116°\ 29'$, $\gamma=88°\ 9'$. In most cases there is a close resemblance to orthoclase (p. 56), but the inclination of the axis of B gives a slight slope to the edge between 001 and $\bar{1}01$ (*Fig.* 77). In the anorthoclase of the rhomb-porphyries the pinakoids are wanting and the crystal consists of the two hemi-prisms $\{110\}$ and $\{1\bar{1}0\}$ and the hemi-macrodome $\{\bar{2}01\}$, as shown in *Fig.* 78.

Axinite. a : b : c=0.49211 : 1 : 0.47970; $\alpha=82°\ 54'$, $\beta=91°\ 52'$, $\gamma=131°\ 32'$. The hemi-prism $\{1\bar{1}0\}$ and quarter-pyramid $\{1\bar{1}1\}$ often show the largest faces present, while $\{1\bar{1}0\}$, $\{100\}$, $\{201\}$ and $\{111\}$ are less well developed (*Fig.* 79).

Kyanite. $a : b : c = 0.89938 : 1 : 0.70896$; $\alpha = 90^\circ\ 5'$, $\beta = 101^\circ\ 2'$, $\gamma = 105^\circ\ 44'$. The long bladed crystals are rarely terminated. The macropinakoid $\{100\}$, with its perfect cleavage, is the most prominent form, and one or more hemi-prisms, $\{110\}$, $\{210\}$, $\{1\bar{1}0\}$, may be present, as well as the brachypinakoid $\{010\}$.

CHAPTER IX.

TETRAGONAL SYSTEM—ZIRCON CLASS
(IV Dc, DITETRAGONAL CENTRAL).

In the tetragonal system the three axes are all at right angles, as in the orthorhombic system, but the two horizontal (or lateral) axes are alike in all respects. They have the same parameter, a, which is taken as unity, while the parameter of the vertical (or principal) axis, c, may be either greater or less than unity. The only element therefore of a tetragonal mineral is c, the vertical parameter.

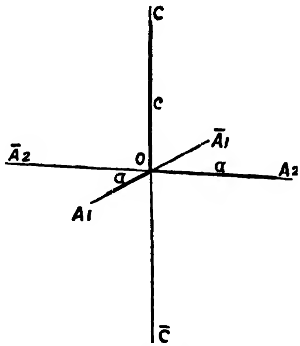


FIG. 80.—Tetragonal Axes.

It is usual to indicate the lateral axes as OA_1 and OA_2, to show their equivalence, the former being the front and back axis (*Fig.* 80).

The zircon class of the tetragonal system possesses a centre of symmetry, one axis of quarter-turn and four of half-turn symmetry, and five planes of symmetry. In addition to the three axial planes of symmetry there are two

diagonal planes of symmetry, which bisect the angles between the vertical axial planes (*Fig.* 81).

The principal axis is one of quarter-turn symmetry, and four half-turn axes lie at the intersections of the four vertical planes of symmetry with the horizontal one. Two of these are parallel to the horizontal crystallographic axes and two are diagonally placed so as to bisect the angles between the others.

The unit pyramid {111} is a form with eight faces as in the orthorhombic system, but in this system they are isosceles, not scalene triangles (*Fig.* 82). The diagonal

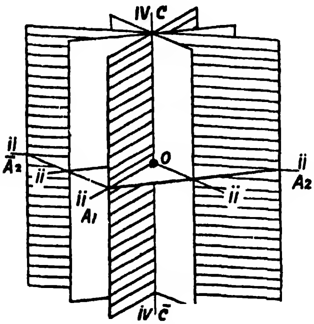


FIG. 81.—Zircon Class. Symmetry, C, 5P, iv., 4ii.

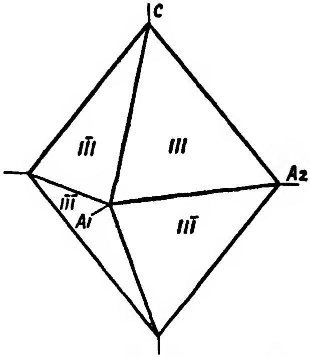


FIG. 82.—Pyramid of the First Order.

planes of symmetry are perpendicular to the faces of this form, and therefore do not give rise to additional faces; nor do the diagonal lines of symmetry; and the conditions of the quarter-turn axis are fulfilled, since the intercepts on the lateral axes are equal. Other forms, such as {112}, {331} and {hhl}, which make equal intercepts on the two lateral axes, also consist of eight faces, which are isosceles triangles. In all these pyramids the horizontal edges form a square, not a rhomb as in the orthorhombic system. They are called pyramids of the first order. The general form {hhl} consists of the faces—

$$h\,h\,l,\quad \bar{h}\,h\,l,\quad \bar{h}\,\bar{h}\,l,\quad h\,\bar{h}\,l,$$
$$h\,h\,\bar{l},\quad \bar{h}\,h\,\bar{l},\quad \bar{h}\,\bar{h}\,\bar{l},\quad h\,\bar{h}\,\bar{l}.$$

The face 211, however, is inclined to a diagonal plane of symmetry. Symmetry therefore requires a corresponding face 121 equally inclined to this plane. The axial plane of symmetry A_1OC requires two similar faces, $2\bar{1}1$ and $1\bar{2}1$ in the top, left octant, and in the same way each octant must be occupied by two faces. The form {211} or {121} has therefore sixteen faces; it is known as a **ditetragonal pyramid** (*Fig.* 83). Similarly, {122}, {231}, or any form $\{h_2 h_1 l\}$, where the two lateral intercepts

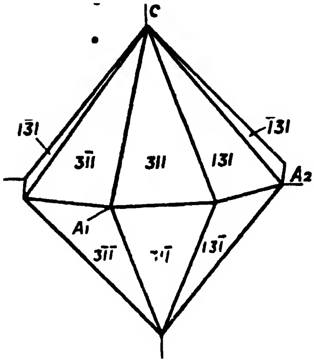


FIG. 83.—Ditetragonal Pyramid.

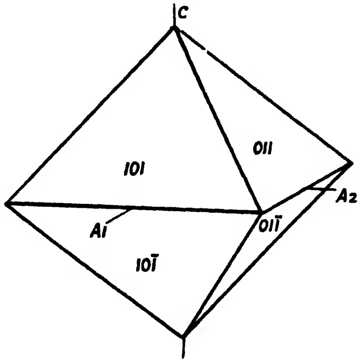


FIG. 84.—Pyramid of the Second Order.

are different, is a ditetragonal pyramid of sixteen faces. The faces of the general form are

$h_2\,h_1\,l$,	$h_1\,h_2\,l$,	$\bar{h}_1\,h_2\,l$,	$\bar{h}_2\,h_1\,l$,
$\bar{h}_2\,\bar{h}_1\,l$,	$\bar{h}_1\,\bar{h}_2\,l$,	$h_1\,\bar{h}_2\,l$,	$h_2\,\bar{h}_1\,l$,
$h_2\,h_1\,\bar{l}$,	$h_1\,h_2\,\bar{l}$,	$\bar{h}_1\,h_2\,\bar{l}$,	$\bar{h}_2\,h_1\,\bar{l}$,
$\bar{h}_2\,\bar{h}_1\,\bar{l}$,	$\bar{h}_1\,\bar{h}_2\,\bar{l}$,	$h_1\,\bar{h}_2\,\bar{l}$,	$h_2\,\bar{h}_1\,\bar{l}$.

It will be seen that not only is there every possible permutation in sign as in the orthorhombic system, but the first two indices are interchangeable. They are therefore indicated by the same letter and distinguished only by a number.

The face 101, parallel to one lateral axis and cutting the other two axes, must be accompanied by the three faces $\bar{1}01$, $\bar{1}0\bar{1}$ and $10\bar{1}$ to satisfy the axial planes of symmetry or

the axial lines of symmetry. But each of these four faces is inclined to a diagonal plane of symmetry, necessitating the four additional faces 011, $0\bar{1}1$, $0\bar{1}\bar{1}$, $01\bar{1}$. The quarter-turn axis or the diagonal half-turn axes would give the same result. The form $\{101\}$ therefore consists of eight faces (*Fig.* 84), which are isosceles triangles and form a square pyramid differing in slope (for the same parameters) from the pyramid of the first order, $\{111\}$. It is equivalent to the two domes of the orthorhombic system, and is known as a *pyramid of the second order*. Flatter or steeper pyramids of the second order are represented by

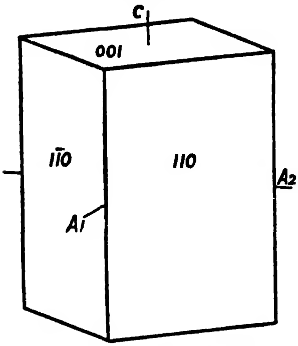


FIG. 85.—Prism of the First Order.

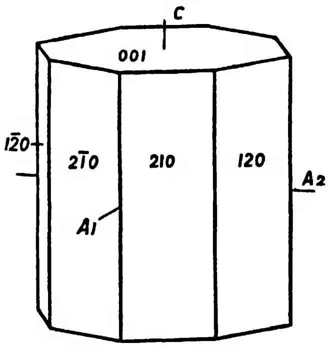


FIG. 86.—Ditetragonal Prism.

$\{102\}$, $\{201\}$, or the general form $\{h0l\}$. The faces of this general form are

$$h0l,\ 0hl,\ \bar{h}0l,\ 0\bar{h}l,$$
$$h0\bar{l},\ 0h\bar{l},\ \bar{h}0\bar{l},\ 0\bar{h}\bar{l}.$$

The face 110, parallel to the vertical axis and with equal intercepts on the lateral axes, is perpendicular to a diagonal plane and line of symmetry, and so is the corresponding face $1\bar{1}0$. The only effective symmetry therefore is the same as in the orthorhombic system, the quarter-turn axis being satisfied by the equality of the intercepts on the lateral crystallographic axes. Hence $\{110\}$ is a form with four faces (*Fig.* 85), as in the orthorhombic system, but its base is a square, not a rhomb. It is called the *prism of*

the first order, as it corresponds to the pyramid of the first order, and its faces are—

$$110,\ \bar{1}10,\ \bar{1}\bar{1}0,\ 1\bar{1}0.$$

If a face, as 210, is parallel to the vertical axis but has unequal intercepts on the two lateral axes, a diagonal plane of symmetry gives rise to 120 also; and these two faces will be repeated with all possible changes of sign, and of position so far as regards the lateral axes. A form with eight vertical faces is the result (*Fig.* 86). This is known as

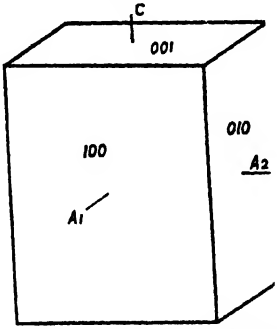


FIG. 87.—Prism of the Second Order.

a ditetragonal prism, and the general form $\{h_2 h_1 0\}$ consists of the eight faces—

$$h_2 h_1 0,\quad h_1 h_2 0,\quad \bar{h}_1 h_2 0,\quad \bar{h}_2 h_1 0,$$
$$\bar{h}_2 \bar{h}_1 0,\quad \bar{h}_1 \bar{h}_2 0,\quad h_1 \bar{h}_2 0,\quad h_2 \bar{h}_1 0.$$

The face 100 must be accompanied by the opposite face $\bar{1}00$ to satisfy the front and back axial plane of symmetry, as well as the centre of symmetry and two of the axial lines of symmetry (*Fig.* 81). The diagonal planes or lines of symmetry, or the quarter-turn axis, will necessitate the faces 010 and $0\bar{1}0$ also. The form {100} is therefore a square-based prism identical with {110} in everything except orientation with regard to the axes. This is known as the prism of the second order. It is shown in *Fig.* 87 in combination with {001}, and its faces are—

$$100,\ 010,\ \bar{1}00,\ 0\bar{1}0.$$

The face 001 must be accompanied by the opposite face $00\bar{1}$. This satisfies the centre of symmetry, horizontal plane of symmetry, and four horizontal lines of symmetry. The form {001} therefore consists of these two faces only. It is known as the basal pinakoid, and is seen in *Figs*. 85 to 87.

The forms occurring in the zircon class of the tetragonal system may be tabulated as follows:—

Name of form.	General symbol.	Example.	Number of faces.
Ditetragonal pyramid	$\{h_2 h_1 l\}$	{211}	16
Pyramid of the first order	{hhl}	{111}	8
Pyramid of the second order	{hol}	{101}	8
Ditetragonal prism	$\{h_2 h_1 o\}$	{210}	8
Prism of the first order	{110}		4
Prism of the second order	{100}		4
Basal pinakoid	{001}		2

It will be seen from the table that the additional symmetry of this class, as compared with the olivine class of the orthorhombic system, is marked by several forms having more faces than the corresponding forms in the latter class. Thus the forms $\{h_2 h_1 l\}$, {hol}, $\{h_2 h_1 o\}$ and {100} have each twice as many faces as the corresponding forms in the olivine class. The forms that have the same number of faces in the two classes are those that have equal intercepts on the two lateral axes, namely, {hhl}, {110} and {001}.

The position of the vertical axis in the zircon class is easily recognised, as it is an axis of quarter-turn symmetry, and differs in its parameter from the other two, which are alike; but the question as to which pair of horizontal half-turn axes is selected as lateral crystallographic axes is purely arbitrary, so long, of course, as those at right angles to each other are selected. In general, however, the selection is made in such a manner that well-developed pyramids and prisms belong to the first order rather than the second, or, if a pyramid of one order is combined with a prism of the other, so that the pyramid is of the first order (*Fig*. 93).

PRACTICAL WORK.

Zircon. c=0.6404. A long or short prism of the first order, {110}, terminated by the corresponding pyramid {111}, is a common type. Sometimes a steeper pyramid {331} and a ditetragonal pyramid {311} are also present, as in *Fig.* 88.

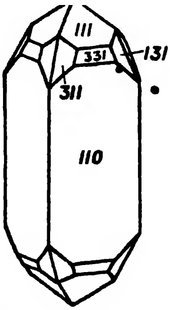


FIG. 88.—Zircon.

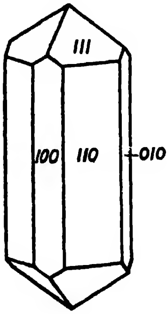


FIG. 89.—Rutile

Rutile. c=0.6442. Rutile closely resembles zircon in angle and habit, but the prism of the second order, {100}, is common (*Fig.* 89), and is sometimes accompanied by the corresponding pyramid.

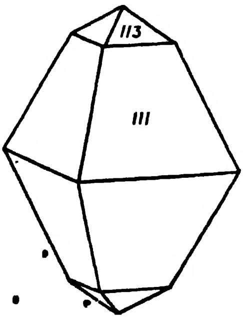


FIG. 90.—Anatase.

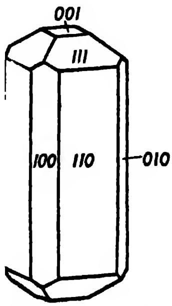


FIG. 91.—Vesuvianite.

Anatase. c=1.7771. As the high value of c indicates, the unit pyramid {111} is steep. This and the flatter pyramid {113}, with the basal pinakoid {001}, are the commonest forms, prisms being rare (*Fig.* 90).

Vesuvianite (Idocrase). c=0.5372. A common type shows the prism {110} and pyramid {111} of the first order with the basal pinakoid {001}. In some examples the second-order prism {100} is well developed (*Fig.* 91).

Melilite. c=0.4548. Usually shows short prisms of the second order {100}, sometimes with first-order and ditetragonal prisms, {110} and {310}, terminated by the basal pinakoid {001}, and occasionally the pyramid of the first order {111}.

Chalcopyrite (Copper Pyrites). c=0.9852. This mineral crystallises not in the zircon or ditetragonal central class, but in the ditetragonal inverse class, IV Dv (see p. 127). It commonly shows a sphenoidal form closely approximating to the tetrahedron (*Fig.* 92). The four faces of the {111} sphenoid are large, dull and striated, while the {1$\bar{1}$1} faces, if present, are small, brilliant and not striated.

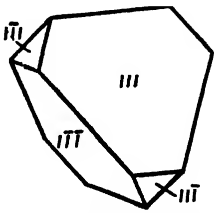


FIG. 92.—Chalcopyrite.

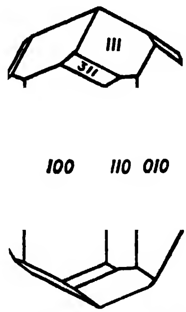


FIG. 93.—Scapolite.

Scapolite. c=0.4384 (for Wernerite). Prisms of the second and first order, {100} and {110}, with the pyramid of the first order, {111} are usually seen. Sometimes the presence of eight small faces of the {311} form, as in *Fig.* 93, indicates the absence of the vertical planes of symmetry, and therefore a lower symmetry than that of the zircon class, with only a quarter-turn axis, one plane and a centre of symmetry. This is the symmetry of the tetragonal central class, IV Mc (see p. 127).

CHAPTER X.

CUBIC SYSTEM—SPINEL CLASS
(4 III Dc, TETRA-DITRIGONAL CENTRAL).

The cubic system is characterised by three axes at right angles and with equal parameters. There are no variable elements, and the angles of one cubic mineral are identical with those of the corresponding forms of all other cubic minerals.

The spinel class possesses a high degree of symmetry, and this gives rise to forms with a large number of faces,

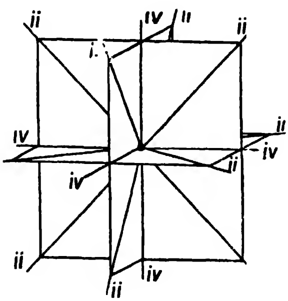


Fig. 94.—Spinel Class, Showing c, 3p, 3 iv, 6 ii.

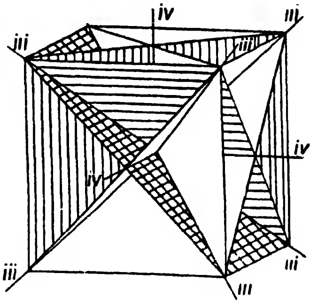


Fig. 95.—Spinel Class, Showing 6p, 3 iv, 4 iii.

from six to forty-eight. There are nine planes of symmetry, three axes of quarter-turn, six of half-turn, and four of one-third-turn symmetry, and a centre of symmetry. Three of the planes of symmetry are axial planes (*Fig.* 94), and six are diagonal planes (*Fig.* 95). The axial planes intersect in three axes of quarter-turn symmetry (iv. in *Figs.* 94 and 95), which are the crystallographic axes. Two axes of half-turn symmetry lie in each of the three axial planes, making angles of 45° with the crystallographic axes (ii. in

Fig. 94). One axis of one-third-turn symmetry passes through the centre of each pair of opposite octants, equidistant from the three crystallographic axes; it is the line of intersection of three diagonal planes of symmetry (iii. in *Fig.* 95).

As a result of the axial planes of symmetry, the face 111 must be accompanied by seven similar faces—$\bar{1}11$, $\bar{1}\bar{1}1$, $1\bar{1}1$, $11\bar{1}$, $\bar{1}1\bar{1}$, $\bar{1}\bar{1}\bar{1}$, and $1\bar{1}\bar{1}$—and these eight faces satisfy the conditions of all the axes, the diagonal planes, and the centre of symmetry. Thus the form {111} consists of eight faces, as in the orthorhombic system, but the faces are

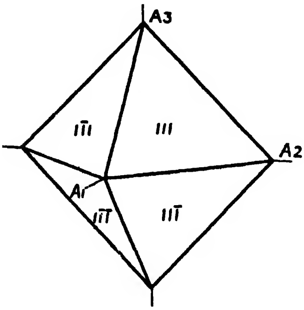


FIG. 96.—Octahedron.

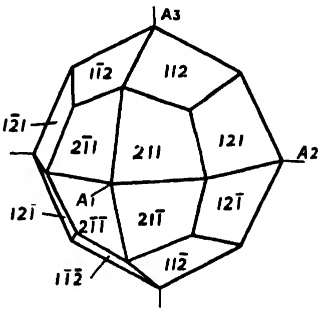


FIG. 97.—Trapezohedron.

equilateral triangles and all the edges are alike, measuring 70° 31′ 44″. It is known as the octahedron (*Fig.* 96).

The face 211 is inclined to two of the three diagonal planes of symmetry that cut the positive octant. It must therefore be accompanied by the faces 121 and 112; and these three faces are repeated in each of the other octants, giving a form with twenty-four faces (*Fig.* 97) whose indices are all the possible permutations of 2, 1, 1, with all possible changes of sign. Since the faces are trapezoids and three of them occur in each octant, in place of one face of the octahedron, this form is called a trapezoidal triakis-octahedron, or more briefly, a trapezohedron.* Any other form with one index

* Also known as a leucitohedron or eikositetrahedron.

greater than the other two, which are equal, such as {322} or $\{h_2 h_1 h_1\}$ where h_2 is greater than h_1, has also twenty-four trapezoidal faces, but with different angles, and is a trapezohedron. The faces of the general form are:—

$$
\begin{array}{lll}
h_2 h_1 h_1, & h_1 h_2 h_1, & h_1 h_1 h_2, \\
\bar{h}_2 h_1 h_1, & \bar{h}_1 h_2 h_1, & \bar{h}_1 h_1 h_2, \\
\bar{h}_2 \bar{h}_1 h_1, & \bar{h}_1 \bar{h}_2 h_1, & \bar{h}_1 \bar{h}_1 h_2, \\
h_2 \bar{h}_1 h_1, & h_1 \bar{h}_2 h_1, & h_1 \bar{h}_1 h_2, \\
h_2 h_1 \bar{h}_1, & h_1 h_2 \bar{h}_1, & h_1 h_1 \bar{h}_2, \\
\bar{h}_2 h_1 \bar{h}_1, & \bar{h}_1 h_2 \bar{h}_1, & \bar{h}_1 h_1 \bar{h}_2, \\
\bar{h}_2 \bar{h}_1 \bar{h}_1, & \bar{h}_1 \bar{h}_2 \bar{h}_1, & \bar{h}_1 \bar{h}_1 \bar{h}_2, \\
h_2 \bar{h}_1 \bar{h}_1, & h_1 \bar{h}_2 \bar{h}_1, & h_1 \bar{h}_1 \bar{h}_2.
\end{array}
$$

In the same way, the face 221 is inclined to two of the three diagonal planes of symmetry in the positive octant,

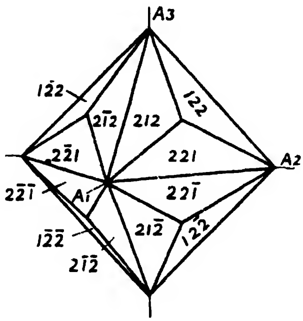


Fig. 98.—Triangular Triakis-Octahedron.

and must be accompanied by the faces 212 and 122; and these faces repeated in the other octants give another form with twenty-four faces. These faces are triangles, and the form {221} is called a triangular triakis-octahedron (*Fig.* 98). Any other form with two equal indices greater than the third, such as {332} or $\{h_2 h_2 h_1\}$ where h_2 is greater than h_1, will also be a triangular triakis-octahedron.

The faces are, for the general form :—

$$\begin{array}{lll}
h_2\,h_2\,h_1, & h_2\,h_1\,h_2, & h_1\,h_2\,h_2, \\
\bar{h}_2\,h_2\,h_1, & \bar{h}_2\,h_1\,h_2, & \bar{h}_1\,h_2\,h_2, \\
\bar{h}_2\,\bar{h}_2\,h_1, & \bar{h}_2\,\bar{h}_1\,h_2, & \bar{h}_1\,\bar{h}_2\,h_2, \\
h_2\,\bar{h}_2\,h_1, & h_2\,\bar{h}_1\,h_2, & h_1\,\bar{h}_2\,h_2, \\
 & & \\
h_2\,h_2\,\bar{h}_1, & h_2\,h_1\,\bar{h}_2, & h_1\,h_2\,\bar{h}_2, \\
\bar{h}_2\,h_2\,\bar{h}_1, & \bar{h}_2\,h_1\,\bar{h}_2, & \bar{h}_1\,h_2\,\bar{h}_2, \\
\bar{h}_2\,\bar{h}_2\,\bar{h}_1, & \bar{h}_2\,\bar{h}_1\,\bar{h}_2, & \bar{h}_1\,\bar{h}_2\,\bar{h}_2, \\
h_2\,\bar{h}_2\,\bar{h}_1, & h_2\,\bar{h}_1\,\bar{h}_2, & h_1\,\bar{h}_2\,\bar{h}_2.
\end{array}$$

The face 321 is inclined to all the three diagonal planes of symmetry in the positive octant. These give rise to the faces 231, 132, 123, 213 and 312. Each octant therefore has six faces, and the form {321} has forty-eight faces (*Fig.* 99).

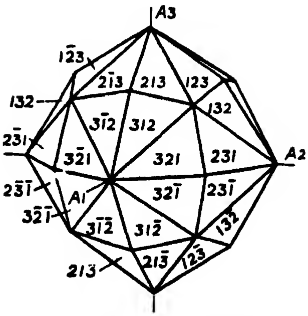


FIG. 99.—Hexakis-Octahedron.

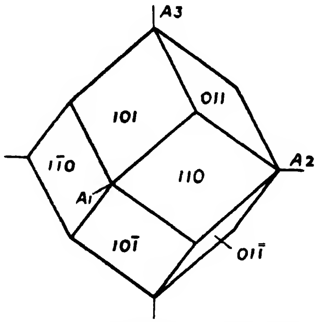


FIG. 100.—Rhombic Dodecahedron.

It is called a h e x a k i s - o c t a h e d r o n (Gr. *hexakis*, six times). The general form is $\{h_3\,h_2\,h_1\}$, *i.e.*, all the indices are different, and the faces are :—

$$\begin{array}{llllll}
h_3\,h_2\,h_1, & h_2\,h_3\,h_1, & h_1\,h_3\,h_2, & h_1\,h_2\,h_3, & h_2\,h_1\,h_3, & h_3\,h_1\,h_2, \\
\bar{h}_3\,h_2\,h_1, & \bar{h}_2\,h_3\,h_1, & \bar{h}_1\,h_3\,h_2, & \bar{h}_1\,h_2\,h_3, & \bar{h}_2\,h_1\,h_3, & \bar{h}_3\,h_1\,h_2, \\
\bar{h}_3\,\bar{h}_2\,h_1, & \bar{h}_2\,\bar{h}_3\,h_1, & \bar{h}_1\,\bar{h}_3\,h_2, & \bar{h}_1\,\bar{h}_2\,h_3, & \bar{h}_2\,\bar{h}_1\,h_3, & \bar{h}_3\,\bar{h}_1\,h_2, \\
h_3\,\bar{h}_2\,h_1, & h_2\,\bar{h}_3\,h_1, & h_1\,\bar{h}_3\,h_2, & h_1\,\bar{h}_2\,h_3, & h_2\,\bar{h}_1\,h_3, & h_3\,\bar{h}_1\,h_2, \\
 & & & & & \\
h_3\,h_2\,\bar{h}_1, & h_2\,h_3\,\bar{h}_1, & h_1\,h_3\,\bar{h}_2, & h_1\,h_2\,\bar{h}_3, & h_2\,h_1\,\bar{h}_3, & h_3\,h_1\,\bar{h}_2, \\
\bar{h}_3\,h_2\,\bar{h}_1, & \bar{h}_2\,h_3\,\bar{h}_1, & \bar{h}_1\,h_3\,\bar{h}_2, & \bar{h}_1\,h_2\,\bar{h}_3, & \bar{h}_2\,h_1\,\bar{h}_3, & \bar{h}_3\,h_1\,\bar{h}_2, \\
\bar{h}_3\,\bar{h}_2\,\bar{h}_1, & \bar{h}_2\,\bar{h}_3\,\bar{h}_1, & \bar{h}_1\,\bar{h}_3\,\bar{h}_2, & \bar{h}_1\,\bar{h}_2\,\bar{h} & \bar{h}_2\,\bar{h}_1\,\bar{h}_3, & \bar{h}_3\,\bar{h}_1\,\bar{h}_2, \\
h_3\,\bar{h}_2\,\bar{h}_1, & h_2\,\bar{h}_3\,\bar{h}_1, & h_1\,\bar{h}_3\,\bar{h}_2, & h_1\,\bar{h}_2\,\bar{h}_3, & h_2\,\bar{h}_1\,\bar{h}_3, & h_3\,\bar{h}_1\,\bar{h}_2.
\end{array}$$

The face 110, which is parallel to one axis and cuts the other two at equal distances, must be accompanied by the faces $\bar{1}10$, $\bar{1}\bar{1}0$ and $1\bar{1}0$, to satisfy the axial planes of symmetry. The diagonal planes of symmetry give rise to the faces 101 and 011, and each of these must have three similar faces with indices showing variations of sign. The form {110} therefore consists of twelve faces, equivalent to a combination of the prism, macrodome and brachydome of the orthorhombic system (*Fig.* 100). The faces are rhombs, and the form is therefore called the rhombic

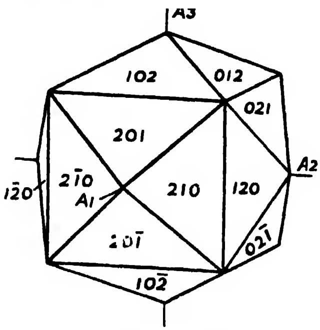


FIG. 101.—Tetrakis-Hexahedron.

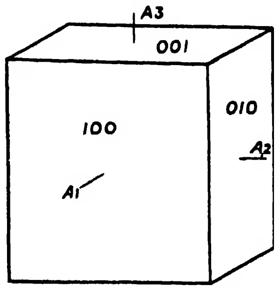


FIG. 102.—Cube.

dodecahedron to distinguish it from other dodecahedra (Gr. *dodeka*, twelve, *hedra*, seat). The symbols of its faces are:—

$$110, \bar{1}10, \bar{1}\bar{1}0, 1\bar{1}0,$$
$$101, \bar{1}01, \bar{1}0\bar{1}, 10\bar{1},$$
$$011, 0\bar{1}1, 0\bar{1}\bar{1}, 01\bar{1}.$$

The face 210 must be accompanied by 120, equally inclined to one of the diagonal planes of symmetry, and each of these faces requires eleven others as shown in the last paragraph. The form {210} therefore has twenty-four faces, as in *Fig.* 101. It resembles a cube with a pyramid erected on each of its faces, and is called a four-faced cube or tetrakis-hexahedron. Any form in this class with faces parallel to one axis and unequally inclined

to the other two is a four-faced cube, and the faces of the general form $\{h_2 h_1 o\}$ are :—

$$\begin{array}{llll} h_2 h_1 o, & h_1 h_2 o, & \bar{h}_1 h_2 o, & \bar{h}_2 h_1 o, \\ \bar{h}_2 \bar{h}_1 o, & \bar{h}_1 \bar{h}_2 o, & h_1 \bar{h}_2 o, & h_2 \bar{h}_1 o, \\ h_2 o h_1, & h_1 o h_2, & \bar{h}_1 o h_2, & \bar{h}_2 o h_1, \\ \bar{h}_2 o \bar{h}_1, & \bar{h}_1 o \bar{h}_2, & h_1 o \bar{h}_2, & h_2 o \bar{h}_1, \\ o h_2 h_1, & o h_1 h_2, & o \bar{h}_1 h_2, & o \bar{h}_2 h_1, \\ o \bar{h}_2 \bar{h}_1, & o \bar{h}_1 \bar{h}_2, & o h_1 \bar{h}_2, & o h_2 \bar{h}_1. \end{array}$$

The face 100, which is parallel to two axes, must be accompanied by the faces 010 and 001 to satisfy the diagonal planes of symmetry, and these must have their opposite faces with indices of opposite sign. The form {100} therefore has six similar faces, and is equivalent to the three pinakoids of the orthorhombic system. It is known as the cube or hexahedron (*Fig.* 102). The faces of the cube are :—

$$\begin{array}{lll} 100, & 010, & 001, \\ \bar{1}00, & 0\bar{1}0, & 00\bar{1}. \end{array}$$

The forms occurring in the spinel class of the cubic system may be tabulated as follows :—

Name of form.	General symbol.		Example.	Number of faces.
Hexakis-octahedron	$\{h_3 h_2 h_1\}$		{321}	48
Triangular triakis-octahedron	$\{h_2 h_2 h_1\}$	$(h_2 > h_1)$	{221}	24
Trapezohedron	$\{h_2 h_1 h_1\}$	$(h_2 > h_1)$	{211}	24
Octahedron	{111}			8
Four-faced cube	$\{h_2 h_1 o\}$	$(h_2 > h_1)$	{210}	24
Rhombic dodecahedron	{110}			12
Cube	{100}			6

All these are closed forms and may exist as simple forms. The octahedron, {111}, has the same number of faces as {111} in the olivine class of the orthorhombic system; the other forms have either three or six times as many faces as the corresponding forms in that class.

In drawing crystals of the spinel class one view only is

sufficient, as the views from the side and from above are the same as the front view. The front elevation of two of the forms in this class is shown in *Figs.* 35 and 36, p. 36.

The following construction will be found useful in drawing a hexakis-octahedron. Two lines are drawn intersecting at right angles, to represent the axes OA_2 and OA_3. The indices of the form are written in ascending order, and the second and third are added together. The lowest common multiple of these four figures is found, and four points are marked on each direction of each axis at distances from

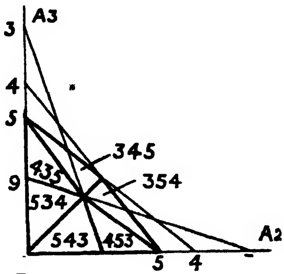


FIG. 103.—Part Construction of Hexakis-Octahedron.

the origin proportional to the reciprocals of these figures. Thus, in drawing the form {543}, we take the figures 3, 4, 5, 9. Their least common multiple is 180. The point 3 is taken at $\frac{1}{3} \times 180 = 60$ mm. from the origin; the point 4 at $\frac{1}{4} \times 180 = 45$ mm.; the point 5 at $\frac{1}{5} \times 180 = 36$ mm.; and the point 9 at $\frac{1}{9} \times 180 = 20$ mm. from the origin. The point on one axis corresponding to the lowest number is joined to those on the other axis corresponding to the highest number and *vice versa*, thus 3 to 9 and 9 to 3, and also the intermediate points in the same manner, 4 to 5 and 5 to 4. The remaining connections are made as in *Fig.* 103, which shows one quarter only of the complete construction. A similar construction may be used for the triakis-octahedra, bearing in mind that two of the four points marked on the axes for the hexakis-octahedron are in these forms merged into one.

Figs. 104 and 105 show part of the construction for the forms {322} and {332}.

The distribution of the indices of the faces of these rather complicated forms is easily found by the use of the rule, "the nearer the axis, the larger the index" (p. 45). Thus in the form {543} (*Fig.* 103) the two faces nearest the front (closest to the axis OA_1) will have the largest index, 5, in the f i r s t place. The face near the plane OA_1A_2 must be nearer to OA_2 than to OA_3; the second index must therefore be greater than the third, and the indices of this face will be 543; while the front face above it near the plane OA_1A_3 will be 534 since it is nearer to OA_3 than to OA_2. Similarly the faces at

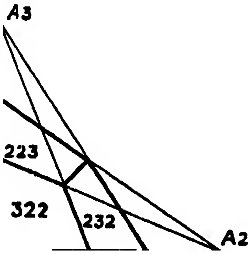


FIG. 104.—Part Construction of Trapezohedron.

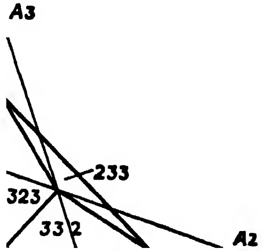


FIG. 105.—Part Construction of Triangular Triakis-Octahedron.

the extreme right or left will have the largest index 5 (or $\bar{5}$) in the s e c o n d place, and those at the top and bottom will have it in the t h i r d place.

It will be noticed that in a hexakis-octahedron each face is the mirror-image of the adjacent faces, but is superposable on the alternate faces. In each octant, therefore, there is a group of three alternate faces which are superposable on one another, and a second group of three faces which are the mirror-image of these. The indices of one group of faces are all in descending cyclic order, *e.g.*, 543, 354, 435, while those of the second group are in ascending cyclic order, 345, 534, 453. This affords a means of checking the symbols.

In the form {322}, the front face in the positive octant is nearest to the axis OA_1, and equidistant from OA_2 and OA_3. Its first index therefore is the larger one, 3, and the second and third indices are the smaller, 2. Similarly the side face is 232, and the top face 223.

In the form {332} one face in the positive octant is equally close to axes OA_1 and OA_2, but more distant from OA_3. Its symbol is therefore 332. The face 233 is that most distant from OA_1, and 323 is that most distant from OA_2.

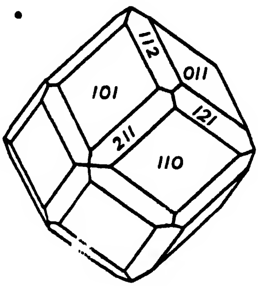


FIG. 106.—Garnet

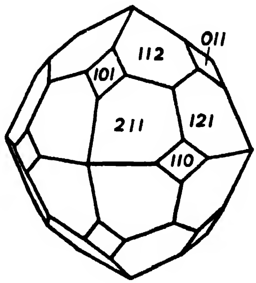


FIG. 107.—Garnet.

It should be noted that an axial plane of symmetry connects faces whose indices differ by one change of sign, and a diagonal plane of symmetry connects those differing in the order of their indices, with or without two changes of sign.

PRACTICAL WORK.

Garnet Group. The rhombic dodecahedron {110} and the trapezohedron {211} are the commonest forms, one or other being dominant, as shown in *Figs.* 106 and 107.

Spinel Group. The dominant form is the octahedron, {111}, either alone, as in magnetite, or modified by {110}, {311}, etc., as in *Fig.* 108.

Leucite. Though not strictly cubic at ordinary temperatures, leucite usually shows crystals not differing sensibly in angle from the trapezohedron {211}, rarely modified by any other form.

Sodalite, Nosean, Haüyne. The rhombic dodecahedron is the commonest form in these minerals. Crystals are sometimes elongated on an axis of threefold symmetry, simulating rhombohedral crystals.

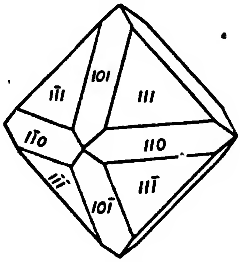


FIG. 108.—Spinel.

Galena, Fluorspar. The cube is the usual form in which these minerals occur, but while the cleavage in galena is parallel to the cube faces, in fluorspar it is octahedral.

CHAPTER XI.

CUBIC SYSTEM (CONTINUED).

TETRAHEDRITE CLASS

(4 III Dc, TETRA-DITRIGONAL UNITERMINAL).

The tetrahedrite class differs from the spinel class in the absence of a centre of symmetry, the three axial planes of symmetry, and the six half-turn axes of symmetry of the spinel class; and in the lower symmetry of the crystallographic axes, which are axes of half-turn, not quarter-turn symmetry. It possesses only the six diagonal planes of

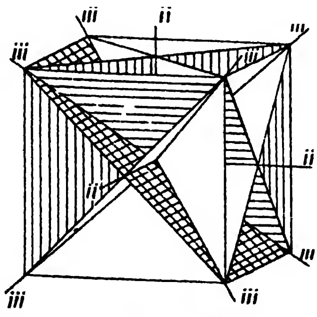


FIG. 109.—Tetrahedrite Class—Symmetry, 6P, 3 ii, 4 iii.

symmetry, three axes of half-turn symmetry, and four axes of one-third-turn symmetry. The last item is the only feature common to all the classes of the cubic system. *Fig.* 109 indicates this symmetry.

The face 111 is at right angles to three of the six planes of symmetry. The other three planes of symmetry give rise to the faces $\bar{1}\bar{1}1$, $\bar{1}1\bar{1}$ and $1\bar{1}\bar{1}$. As there is no centre of symmetry, no axial planes of symmetry, and no half-turn

axes of symmetry, other than the crystallographic axes, the remaining faces of the octahedron are not required by the symmetry of this class, the conditions of which are fulfilled by the four faces named. These faces, if no other form is present, are equilateral triangles meeting in six edges, each

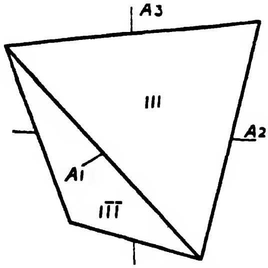


FIG. 110.—Positive Tetrahedron.

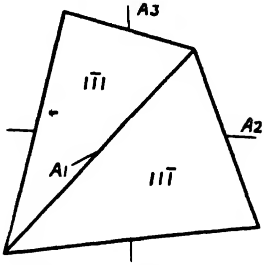


FIG. 111.—Negative Tetrahedron.

of which lies in one of the planes of symmetry (*Fig.* 110). This form {111} is known as a tetrahedron (Gr. *tetra*, four, *hedra*, a seat). The form {1$\bar{1}$1} is similar but differently orientated (*Fig.* 111). It may be called the

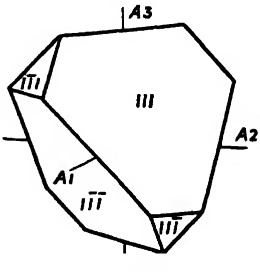


FIG. 112.—Positive and Negative Tetrahedra.

negative tetrahedron to distinguish it from {111}, which is the positive tetrahedron. The two forms often occur together, as in zinc-blende (*Fig.* 112). The faces of the two forms are :—

Positive tetrahedron, 111, $\bar{1}\bar{1}1$, $\bar{1}1\bar{1}$, $1\bar{1}\bar{1}$.

Negative tetrahedron, $1\bar{1}1$, $\bar{1}11$, $\bar{1}\bar{1}\bar{1}$, $11\bar{1}$.

It will be noticed that the product of the indices of each face is positive in the positive tetrahedron, negative in the negative tetrahedron.

The face 211 must be accompanied by the faces 121 and 112 to satisfy the planes of symmetry which cut the positive octant. As in the last paragraph, however, the symmetry of the class only requires that alternate octants shall be similarly occupied. The form {211} therefore has only twelve faces, not twenty-four as in the spinel class. It is shown in *Fig.* 113, from which it will be seen that the faces are isosceles triangles. It is known as a triangular triakis-tetrahedron. In general terms $\{h_2 h_1 h_1\}$,

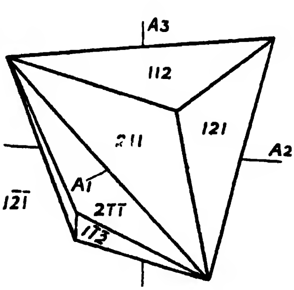


FIG. 113.—Triangular Triakis-Tetrahedron.

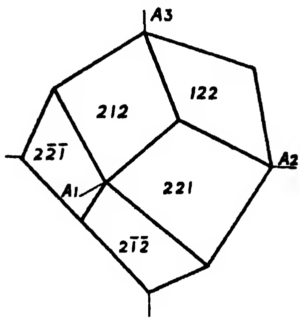


FIG. 114.—Trapezoidal Triakis-Tetrahedron.

where h_2 is greater than h_1, represents the positive triangular triakis-tetrahedron, with the faces :—

$$\begin{array}{lll} h_2 h_1 h_1, & h_1 h_2 h_1, & h_1 h_1 h_2, \\ \bar{h}_2 \bar{h}_1 h_1, & \bar{h}_1 \bar{h}_2 h_1, & \bar{h}_1 \bar{h}_1 h_2, \\ \bar{h}_2 h_1 \bar{h}_1, & \bar{h}_1 h_2 \bar{h}_1, & \bar{h}_1 h_1 \bar{h}_2, \\ h_2 \bar{h}_1 \bar{h}_1, & h_1 \bar{h}_2 \bar{h}_1, & h_1 \bar{h}_1 \bar{h}_2, \end{array}$$

and $\{h_2 \bar{h}_1 h_1\}$ represents the negative form with the faces :—

$$\begin{array}{lll} h_2 \bar{h}_1 h_1, & h_1 \bar{h}_2 h_1, & h_1 \bar{h}_1 h_2, \\ \bar{h}_2 h_1 h_1, & \bar{h}_1 h_2 h_1, & \bar{h}_1 h_1 h_2, \\ \bar{h}_2 \bar{h}_1 \bar{h}_1, & \bar{h}_1 \bar{h}_2 \bar{h}_1, & \bar{h}_1 \bar{h}_1 \bar{h}_2, \\ h_2 h_1 \bar{h}_1, & h_1 h_2 \bar{h}_1, & h_1 h_1 \bar{h}_2. \end{array}$$

The face 221 must be accompanied by the faces 212 and 122 in the positive octant, and these, with similar faces in the alternate octants, give another form with twelve faces (*Fig.* 114). In this form the faces are trapezoids (or deltoids) and the form {221} is known as a trapezoidal (or deltoidal) triakis-tetrahedron. In general terms $\{h_2 h_2 h_1\}$, where h_2 is greater than h_1, represents the positive trapezoidal triakis-octahedron, with the faces

$$
\begin{array}{lll}
h_2 h_2 h_1, & h_2 h_1 h_2, & h_1 h_2 h_2, \\
\bar{h}_2 \bar{h}_2 h_1, & \bar{h}_2 \bar{h}_1 h_2, & \bar{h}_1 \bar{h}_2 h_2, \\
\bar{h}_2 h_2 \bar{h}_1, & \bar{h}_2 h_1 \bar{h}_2, & \bar{h}_1 h_2 \bar{h}_2, \\
h_2 \bar{h}_2 \bar{h}_1, & h_2 \bar{h}_1 \bar{h}_2, & h_1 \bar{h}_2 \bar{h}_2,
\end{array}
$$

and $\{h_2 \bar{h}_2 h_1\}$ represents the negative form with the faces

$$
\begin{array}{lll}
h_2 \bar{h}_2 h_1, & h_2 \bar{h}_1 h_2, & h_1 \bar{h}_2 h_2, \\
\bar{h}_2 h_2 h_1, & \bar{h}_2 h_1 h_2, & \bar{h}_1 h_2 h_2, \\
\bar{h}_2 \bar{h}_2 \bar{h}_1, & \bar{h}_2 \bar{h}_1 \bar{h}_2, & \bar{h}_1 \bar{h}_2 \bar{h}_2, \\
h_2 h_2 \bar{h}_1, & h_2 h_1 \bar{h}_2, & h_1 h_2 \bar{h}_2.
\end{array}
$$

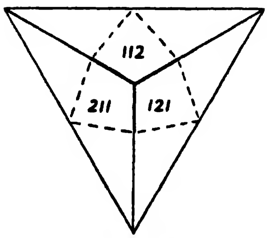


FIG. 115.

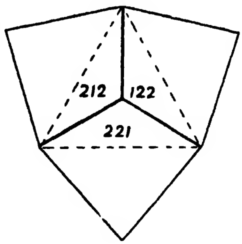


FIG. 116.

The Triakis-Tetrahedra (solid lines) and Triakis-Octahedra (broken lines). Projections on 111.

It will be noticed that the form {211} is a trapezoidal triakis-octahedron in the spinel class and a triangular triakis-tetrahedron in the tetrahedrite class, and that {221} is a triangular triakis-octahedron and a trapezoidal triakis-tetrahedron. The edges in which the three faces in the positive octant intersect, however, remain unchanged in the two classes.

In the {211} forms they resemble the letter Y, or the crest of the Isle of Man, while in the {221} forms they are more like the Greek letter λ or the crest of the Cyclists' Touring Club (see *Figs*. 115 and 116). The same applies to the general forms $\{h_2 h_1 h_1\}$ and $\{h_2 h_2 h_1\}$.

The face 321 must be accompanied by the faces 231, 132, 123, 213 and 312, and these, with six corresponding faces in each alternate octant, give a form {321} composed of twenty-four faces (*Fig*. 117). It is known as a h e x a k i s -

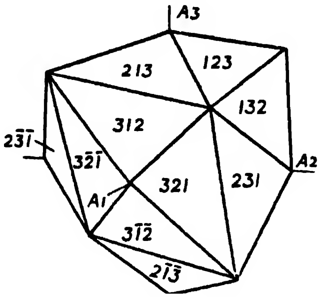


FIG. 117.—Hexakis-Tetrahedron.

t e t r a h e d r o n , and the general form $\{h_3 h_2 h_1\}$ represents the positive hexakis-tetrahedron with the faces

$$\begin{array}{llllll}
h_3 h_2 h_1, & h_2 h_3 h_1, & h_1 h_3 h_2, & h_1 h_2 h_3, & h_2 h_1 h_3, & h_3 h_1 h_2, \\
\bar{h}_3 \bar{h}_2 h_1, & \bar{h}_2 \bar{h}_3 h_1, & \bar{h}_1 \bar{h}_3 h_2, & \bar{h}_1 \bar{h}_2 h_3, & \bar{h}_2 \bar{h}_1 h_3, & \bar{h}_3 \bar{h}_1 h_2, \\
\bar{h}_3 h_2 \bar{h}_1, & \bar{h}_2 h_3 \bar{h}_1, & \bar{h}_1 h_3 \bar{h}_2, & \bar{h}_1 h_2 \bar{h}_3, & \bar{h}_2 h_1 \bar{h}_3, & \bar{h}_3 h_1 \bar{h}_2, \\
h_3 \bar{h}_2 \bar{h}_1, & h_2 \bar{h}_3 \bar{h}_1, & h_1 \bar{h}_3 \bar{h}_2, & h_1 \bar{h}_2 \bar{h}_3, & h_2 \bar{h}_1 \bar{h}_3, & h_3 \bar{h}_1 \bar{h}_2,
\end{array}$$

while the negative form has the faces

$$\begin{array}{llllll}
h_3 \bar{h}_2 h_1, & h_2 h_3 h_1, & h_1 \bar{h}_3 h_2, & h_1 \bar{h}_2 h_3, & h_2 \bar{h}_1 h_3, & h_3 \bar{h}_1 h_2, \\
\bar{h}_3 h_2 h_1, & \bar{h}_2 h_3 h_1, & \bar{h}_1 h_3 h_2, & \bar{h}_1 h_2 h_3, & \bar{h}_2 h_1 h_3, & \bar{h}_3 h_1 h_2, \\
\bar{h}_3 \bar{h}_2 \bar{h}_1, & \bar{h}_2 \bar{h}_3 \bar{h}_1, & \bar{h}_1 \bar{h}_3 \bar{h}_2, & \bar{h}_1 \bar{h}_2 \bar{h}_3, & \bar{h}_2 \bar{h}_1 \bar{h}_3, & \bar{h}_3 \bar{h}_1 \bar{h}_2, \\
h_3 h_2 \bar{h}_1, & h_2 h_3 \bar{h}_1, & h_1 h_3 \bar{h}_2, & h_1 h_2 \bar{h}_3, & h_2 h_1 \bar{h}_3, & h_3 h_1 \bar{h}_2.
\end{array}$$

The face 110 is perpendicular to one of the six planes of symmetry, parallel to another, and inclined to the remaining four. It must therefore be accompanied by the opposite face $\bar{1}\bar{1}0$ and by 101, $10\bar{1}$, 011, $01\bar{1}$ and $1\bar{1}0$, and these latter must have their opposite faces $\bar{1}0\bar{1}$, $\bar{1}01$, $0\bar{1}\bar{1}$, $0\bar{1}1$ and $\bar{1}10$. The form {110} has therefore twelve faces, as in the spinel class, and is the same rhombic dodecadedron (*Fig.* 100, p. 74). The faces of this form are perpendicular to the axial planes, and therefore are not affected by the absence of axial planes of symmetry.

For the same reason the forms {210} and {100} are unchanged by the lower symmetry of this class, and remain the four-faced cube and the cube respectively, as in the spinel class.

PYRITE CLASS

(4 III Mc, TETRA-TRIGONAL CENTRAL).

In this class the diagonal planes of symmetry of the spinel class are wanting, and the crystallographic axes are lines of half-turn symmetry only. The class possesses

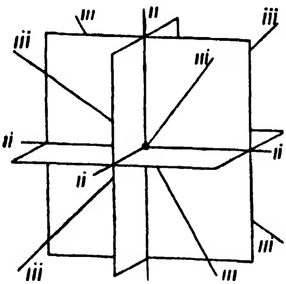


FIG. 118.—Pyrite Class—Symmetry, C, 3P, 3 ii, 4 iii.

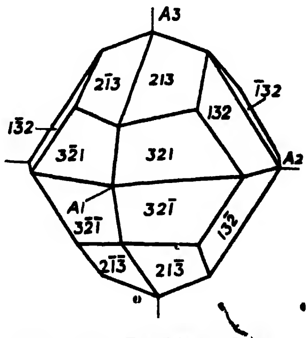


FIG. 119.—Dyakis-Dodecahedron.

therefore a centre of symmetry, three axial planes of symmetry, three axes of half-turn and four of one-third-turn symmetry. In other words, we have here the symmetry of

the olivine class of the orthorhombic system, *plus* the four axes of one-third-turn symmetry, due to the identity in properties of the three crystallographic axes.

The face 111 is repeated in each of the octants by the three axial planes of symmetry, and the form {111} is therefore the octahedron, as in the spinel class.

The face 211 must be accompanied by 121 and 112 to satisfy the trigonal axis in the positive octant, and these faces repeated in the other octants form a trapezohedron, as in the spinel class. Similar reasoning shows that the form {221} is a triangular triakis-octahedron. Thus the general forms $\{h_2 h_1 h_1\}$ and $\{h_2 h_2 h_1\}$ are the same in the pyrite class as in the spinel class.

The face 321 must be accompanied by 132 and 213 about the trigonal axis, but in the absence of the diagonal planes of symmetry the faces with indices in the reverse order (123, 231, 312) are not required. The complete form {321} therefore has $8 \times 3 = 24$ faces instead of 48, as in the spinel class. It is known as the d y a k i s - d o d e c a h e d r o n or d i p l o i d (Gr. *diplos*, double, *eidos*, form), since it has double the number of faces of the pentagonal dodecahedron (see p. 88). *Fig.* 119 shows the form {321}.

If the crystal is turned through 90° about a crystallographic axis it will represent the form {123}, but it is customary to hold this and the following form so that the indices are in descending cyclic order. The faces of the general form $\{h_3 h_2 h_1\}$ are

$h_3 h_2 h_1$,	$h_1 h_3 h_2$,	$h_2 h_1 h_3$,
$\bar{h}_3 h_2 h_1$,	$\bar{h}_1 h_3 h_2$,	$\bar{h}_2 h_1 h_3$,
$\bar{h}_3 \bar{h}_2 h_1$,	$\bar{h}_1 \bar{h}_3 h_2$,	$\bar{h}_2 \bar{h}_1 h_3$,
$h_3 \bar{h}_2 h_1$,	$h_1 \bar{h}_3 h_2$,	$h_2 \bar{h}_1 h_3$,
$h_3 h_2 \bar{h}_1$,	$h_1 h_3 \bar{h}_2$,	$h_2 h_1 \bar{h}_3$,
$\bar{h}_3 h_2 \bar{h}_1$,	$\bar{h}_1 h_3 \bar{h}_2$,	$\bar{h}_2 h_1 \bar{h}_3$,
$\bar{h}_3 \bar{h}_2 \bar{h}_1$,	$\bar{h}_1 \bar{h}_3 \bar{h}_2$,	$\bar{h}_2 \bar{h}_1 \bar{h}_3$,
$h_3 \bar{h}_2 \bar{h}_1$,	$h_1 \bar{h}_3 \bar{h}_2$,	$h_2 \bar{h}_1 \bar{h}_3$.

The face 210 must be accompanied by 021 and 102 about the trigonal axis, but the faces 120, 012 and 201 are not required. The form {210} in the pyrite class has therefore only half the faces of the four-faced cube, *i.e.*, twelve. As shown in *Fig.* 120, the faces are pentagons, but not regular pentagons, those edges which are parallel to the crystallographic axes being longer than the others. This form is known as the pentagonal dodecahedron, or the pyritohedron (*Fig.* 120), from the frequency with which it occurs in pyrite. The faces of the general form $\{h_2 h_1 o\}$ are

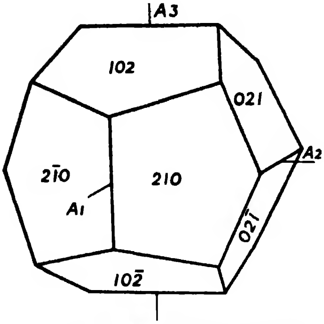


FIG. 120.—Pentagonal Dodecahedron.

$$\begin{array}{llll} h_2 h_1 o, & \bar{h}_2 h_1 o, & \bar{h}_2 \bar{h}_1 o, & h_2 \bar{h}_1 o, \\ o h_2 h_1, & o \bar{h}_2 h_1, & o \bar{h}_2 \bar{h}_1, & o h_2 \bar{h}_1, \\ h_1 o h_2, & \bar{h}_1 o h_2, & \bar{h}_1 o \bar{h}_2, & h_1 o \bar{h}_2. \end{array}$$

The face 110 must be accompanied by 011 and 101 about the trigonal axis, and the axial planes of symmetry bring in faces having the same symbols, but with all possible changes of sign. The form {110} is therefore the rhombic dodecahedron, as in the spinel and tetrahedrite classes.

Similarly the form {100} remains the cube in all three classes.

The different forms in the three classes of the cubic system that have been dealt with are shown in the following table, with the number of faces in each form.

Symbol.	Spinel Class.	Tetrahedrite Class.	Pyrite Class.
$\{h_3 h_2 h_1\}$ *e.g.*, {321}	Hexakis-octahedron, 48.	Hexakis-tetrahedron, 24.	Dyakis-dodecahedron, 24.
$\{h_2 h_2 h_1\}$ *e.g.*, {221}	Triangular triakis-octahedron, 24.	Trapezoidal triakis-tetrahedron, 12.	Triangular triakis-octahedron, 24.

Symbol.	Spinel Class.	Tetrahedrite Class.	Pyrite Class.
$\{h_2 h_1 h_1\}$ *e.g.*, $\{211\}$	Trapezohedron, 24.	Triangular tri-akis-tetrahedron, 12.	Trapezohedron, 24.
$\{111\}$	Octahedron, 8.	Tetrahedron, 4.	Octahedron, 8.
$\{h_2 h_1 0\}$ *e.g.*, $\{210\}$	Four - faced cube, 24.	Four - faced cube, 24.	Pentagonal do-decahedron, 12.
$\{110\}$	Rhombic do-decahedron, 12.	Rhombic do-decahedron, 12.	Rhombic do-decahedron, 12.
$\{100\}$	Cube, 6.	Cube, 6.	Cube, 6.

PRACTICAL WORK.

In the tetrahedrite and pyrite classes three views should be drawn, since the side elevation and ground plan are not identical with the front elevation, as they are in the spinel class.

Zinc-blende. This is the commonest mineral crystallising in the tetrahedrite class. The two tetrahedra may be

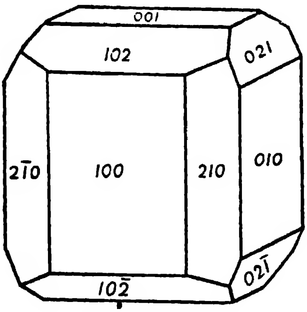


FIG. 121.—Pyrite.

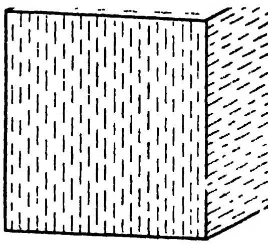


FIG. 122.—Pyrite.

present (*Fig.* 112), with the faces of $\{1\bar{1}1\}$ smaller and duller than those of $\{111\}$, which are large, brilliant and striated. More rarely a triakis-tetrahedron occurs.

Diamond also belongs to this class.

Pyrite. The pentagonal dodecahedron $\{210\}$, the striated cube $\{100\}$ and the octahedron $\{111\}$ are commonly

seen, usually as simple forms, sometimes modified (*Fig.* 121). The cube shows the low symmetry of the class in the striations on its faces (*Fig.* 122), which are parallel to the three pairs of longer edges in the {210} form, and are due to an oscillatory combination of {100} and {210}.

The cobalt minerals, *smaltite* and *cobaltite*, also show beautiful crystals of this class.

CHAPTER XII.

HEXAGONAL SYSTEM—BERYL CLASS
(VI Dc, DIHEXAGONAL CENTRAL).

The hexagonal system is divided into two by some writers, only those classes which have an axis of one-sixth-turn symmetry being placed in the hexagonal, while those with an axis of one-third-turn symmetry are placed in a trigonal or rhombohedral system. The distinction is convenient for many purposes, but if the same crystallographic axes are employed for both groups they may be treated as one system.

Miller referred hexagonal crystals to three axes equally inclined to one another, but not at right angles, and having equal parameters. They are parallel to the edges of the negative rhombohedron (*Fig.* 136), the symbol of which becomes {100}. The corresponding positive rhombohedron (*Fig.* 135) would then be {2$\bar{1}$2}, and the hexagonal pyramid (*Fig.* 127) has faces corresponding to these two forms alternately. The confusion arising from simple forms with two unlike sets of indices is avoided by the use of the four axes proposed by Bravais. These axes are now more usually adopted, although many crystallographers still follow Miller.

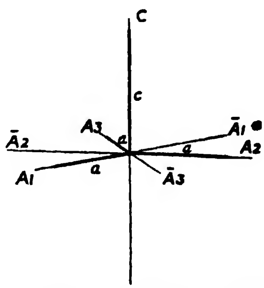


FIG. 123.—Hexagonal Axes.

Of Bravais' axes, three are horizontal and inclined at

120° to one another, and the fourth is vertical (*Fig.* 123). The three horizontal (or lateral) axes have equal parameters, a, which is taken as unity, while the vertical (or principal) axis has a different parameter, c. This vertical parameter is the only variable element in which hexagonal minerals differ from one another; it may be greater or less than unity. Symbols of hexagonal crystals consist of four indices, the first relating to the axis of A_1, the second to that of A_2, the third to that of A_3, and the fourth to the vertical axis of C.

As shown in *Fig.* 123, the axis of A_2 is taken as running right and left, with its positive direction to the right. The positive direction of the axis of A_1 is 120° from that of A_2 in a clockwise direction and points forward to the left of the observer, and the positive direction of the axis of A_3 is at 120° from that of A_2 in a counter-clockwise direction. The negative directions of the three axes thus bisect the angles of 120° between their positive directions.

Fig. 124 represents the three lateral axes, OA_1, OA_2, OA_3, with the trace BDC of a plane $h_1\, h_2\, \bar{h}_3\, l$ that cuts all three of them. The intercepts of this plane on the axes of A_1, A_2 and A_3 are OB, OC and OD respectively, and OD is negative. Draw DE parallel to BO, making ODE an equilaterial triangle. Then

$$\frac{OC}{OB} = \frac{EC}{ED} = \frac{OC - OD}{OD}$$

$$\text{Dividing by OC,} \qquad \frac{1}{OB} = \frac{1}{OD} - \frac{1}{OC}$$

$$\text{or} \qquad \frac{1}{OB} + \frac{1}{OC} = \frac{1}{OD}$$

But the reciprocals of the intercepts are the indices $h_1\, h_2\, h_3$ of the face $h_1\, h_2\, \bar{h}_3\, l$, whose trace on the plane of the lateral axes is BDC, so that $h_1 + h_2 = h_3$. Therefore the third index is equal to the sum of the other two, and is of opposite sign. In other words, the sum of the three lateral indices is always equal to zero.

In the beryl class the vertical axis is one of hexagonal symmetry; the three horizontal axes are lines of digonal symmetry, and three more such lines lie midway between them; there is a centre of symmetry and one horizontal and six vertical planes of symmetry. This symmetry is represented in *Fig.* 125.

A face parallel to all three lateral axes and cutting the vertical axis above the origin will have the symbol 0001. It is affected by the centre of symmetry, the horizontal plane of symmetry and the axes of half-turn symmetry, all of which require a corresponding face $000\bar{1}$ below. The remaining planes and line of symmetry are perpendicular to

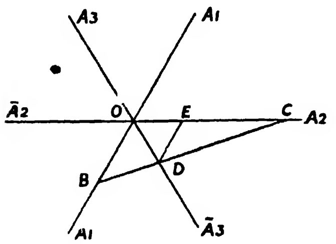


FIG. 124.

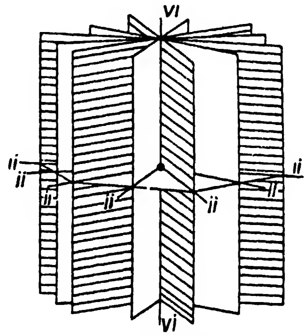


FIG. 125 —Beryl Class—Symmetry, C, 7P, VI, 6ii.

these faces and so inoperative, and the form $\{0001\}$ therefore consists of these two faces only. It is known as the b a s a l p i n a k o i d, and is shown in *Figs.* 126, 128 and 130.

A face parallel to the vertical axis and to one of the lateral axes, such as $10\bar{1}0$, must be accompanied by $01\bar{1}0$, $\bar{1}100$, $\bar{1}010$, $0\bar{1}10$ and $1\bar{1}00$ to satisfy the axis of hexagonal symmetry. These six faces satisfy all the other elements of symmetry of the class, and they therefore constitute the form $\{10\bar{1}0\}$ shown in *Fig.* 126. This is known as the p r i s m o f t h e f i r s t o r d e r. It will be noticed

that the first three indices of the faces are in ascending and descending cyclic order alternately.

A face parallel to one of the lateral axes but inclined to the vertical axis, such as $10\bar{1}1$, must have five similar faces disposed around the axis of hexagonal symmetry, and the horizontal plane of symmetry requires six more faces below. The form $\{10\bar{1}1\}$ has therefore twelve faces, as shown in *Fig.* 127. It is an example of the pyramid of the first order, of which the general form $\{h\,o\,\bar{h}\,l\}$ has the faces

$$h o \bar{h} l,\quad o h \bar{h} l,\quad \bar{h} h o l,\quad \bar{h} o h l,\quad o \bar{h} h l,\quad h \bar{h} o l,$$
$$h o \bar{h} \bar{l},\quad o h \bar{h} \bar{l},\quad \bar{h} h o \bar{l},\quad \bar{h} o h \bar{l},\quad o \bar{h} h \bar{l},\quad h \bar{h} o \bar{l}.$$

A face making equal intercepts on A_1 and A_2 will have half that intercept on the negative end of A_3 (p. 92), and if

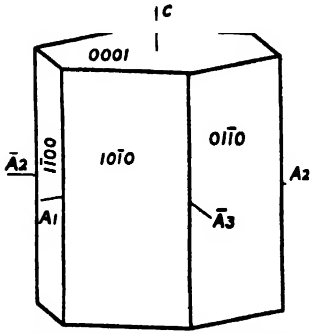


FIG. 126.—Prism of the First Order.

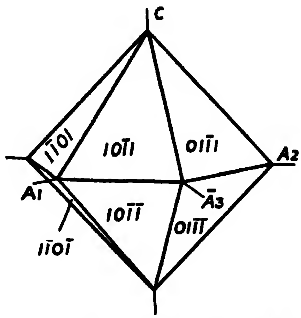


FIG. 127.—Pyramid of the First Order.

it is parallel to the vertical axis its symbol will be $11\bar{2}0$. The hexagonal axis requires the faces $\bar{1}2\bar{1}0$, $\bar{2}110$, $\bar{1}\bar{1}20$, $1\bar{2}10$ and $2\bar{1}\bar{1}0$, and these six faces satisfy the remaining symmetry of the class. The form $\{11\bar{2}0\}$ therefore consists of these six faces. It is known as the prism of the second order, and is shown in *Fig.* 128, which represents a similar prism to *Fig.* 126 rotated 30° about the principal axis.

A face with equal intercepts on the axes of A_1 and A_2, but cutting the vertical axis, such as $11\bar{2}1$, must be accom-

panied by eleven other faces to satisfy the symmetry. The form $\{11\bar{2}1\}$ therefore has twelve faces, and is shown in

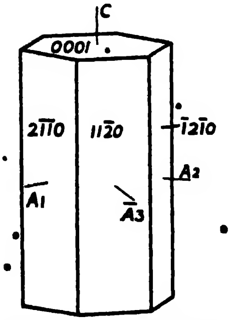


FIG. 128.—Prism of the Second Order.

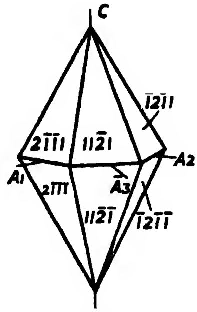


FIG. 129.—Pyramid of the Second Order.

Fig. 129. It is an example of the pyramid of the second order, the general form of which $\{hh\overline{2h}l\}$, consists of the twelve faces

$$
\begin{array}{lll}
h\,h\,\overline{2h}\,l, & \bar{h}\,2h\,\bar{h}\,l, & \overline{2h}\,h\,h\,l, \\
\bar{h}\,\bar{h}\,2h\,l, & h\,\overline{2h}\,h\,l, & 2h\,\bar{h}\,\bar{h}\,l, \\
h\,h\,\overline{2h}\,\bar{l}, & \bar{h}\,2h\,\bar{h}\,\bar{l}, & \overline{2h}\,h\,h\,\bar{l}, \\
\bar{h}\,\bar{h}\,2h\,\bar{l}, & h\,\overline{2h}\,h\,\bar{l}, & 2h\,\bar{h}\,\bar{h}\,\bar{l}.
\end{array}
$$

A face unequally inclined to all three lateral axes, but parallel to the vertical axis, such as $12\bar{3}0$, must be accom-

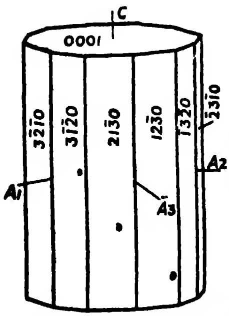


FIG. 130.—Dihexagonal Prism.

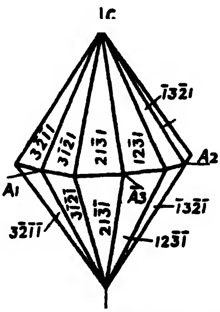


FIG. 131.—Dihexagonal Pyramid.

panied by $21\bar{3}0$ to satisfy one of the vertical planes of symmetry. These two faces repeated round the hexagonal

axis give a prism of twelve faces, which satisfies the symmetry of the class. This form $\{12\bar{3}0\}$ is shown in *Fig.* 130. It is known as a dihexagonal prism, and the general form $\{h_1 h_2 \bar{h}_3 o\}$, where $h_3 = h_1 + h_2$, has the twelve faces

$h_1 h_2 \bar{h}_3 o$,	$\bar{h}_1 h_3 \bar{h}_2 o$,	$\bar{h}_2 h_3 \bar{h}_1 o$,
$\bar{h}_3 h_2 h_1 o$,	$\bar{h}_3 h_1 h_2 o$,	$\bar{h}_2 \bar{h}_1 h_3 o$,
$\bar{h}_1 \bar{h}_2 h_3 o$,	$h_1 \bar{h}_3 h_2 o$,	$h_2 \bar{h}_3 h_1 o$,
$h_3 \bar{h}_2 \bar{h}_1 o$,	$h_3 \bar{h}_1 \bar{h}_2 o$,	$h_2 h_1 \bar{h}_3 o$.

It will be noticed that the first three indices are in ascending and descending cyclic order, h_1, h_2, h_3 and h_3, h_2, h_1, alternately.

The form $\{12\bar{3}1\}$ will have double the number of faces present in the corresponding prism $\{12\bar{3}0\}$, *i.e.*, twenty-four. It is known as a dihexagonal pyramid, and is shown in *Fig.* 131. The general form $\{h_1 h_2 \bar{h}_3 1\}$, where $h_3 = h_1 + h_2$, has the faces

$h_1 h_2 \bar{h}_3 1$,	$\bar{h}_1 h_3 \bar{h}_2 1$,	$\bar{h}_2 h_3 \bar{h}_1 1$,	$\bar{h}_3 h_2 h_1 1$,
$\bar{h}_3 h_1 h_2 1$,	$\bar{h}_2 \bar{h}_1 h_3 1$,	$\bar{h}_1 \bar{h}_2 h_3 1$,	$h_1 \bar{h}_3 h_2 1$,
$h_2 \bar{h}_3 h_1 1$,	$h_3 \bar{h}_2 \bar{h}_1 1$,	$h_3 \bar{h}_1 \bar{h}_2 1$,	$h_2 h_1 \bar{h}_3 1$,
$h_1 h_2 \bar{h}_3 \bar{1}$,	$\bar{h}_1 h_3 \bar{h}_2 \bar{1}$,	$\bar{h}_2 h_3 \bar{h}_1 \bar{1}$,	$\bar{h}_3 h_2 h_1 \bar{1}$,
$\bar{h}_3 h_1 h_2 \bar{1}$,	$\bar{h}_2 \bar{h}_1 h_3 \bar{1}$,	$\bar{h}_1 \bar{h}_2 h_3 \bar{1}$,	$h_1 \bar{h}_3 h_2 \bar{1}$,
$h_2 \bar{h}_3 h_1 \bar{1}$,	$h_3 \bar{h}_2 \bar{h}_1 \bar{1}$,	$h_3 \bar{h}_1 \bar{h}_2 \bar{1}$,	$h_2 h_1 \bar{h}_3 \bar{1}$.

The forms occurring in the beryl class of the hexagonal system may be tabulated as follows:—

Name of form.	General symbol.	Example.	Number of faces.
Dihexagonal pyramid	$\{h_1 h_2 \bar{h}_3 1\}$	$\{12\bar{3}0\}$	24
Pyramid of the second order	$\{h h \overline{2h} 1\}$	$\{11\bar{2}1\}$	12
Pyramid of the first order	$\{h o \bar{h} 1\}$	$\{10\bar{1}1\}$	12
Dihexagonal prism	$\{h_1 h_2 \bar{h}_3 o\}$	$\{12\bar{3}0\}$	12
Prism of the second order	$\{11\bar{2}0\}$		6
Prism of the first order	$\{10\bar{1}0\}$		6
Basal pinakoid	$\{0001\}$		2

In the beryl class the question as to which three half-turn axes making equal angles with one another are taken as lateral axes is an arbitrary one, but it is usual to select them in such a manner that the commoner pyramids and prisms are of the first order.

In drawing hexagonal crystals it is best to start with the view from above, which usually includes a regular hexagon. From this it is easy to obtain construction lines for the front and side elevations. *Fig.* 132 shows these con-

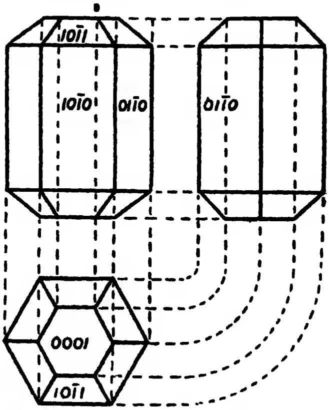


FIG. 132.—Orthographic Projection of Apatite.

struction lines for a combination of {10$\bar{1}$0}, {10$\bar{1}$1} and {0001}.

In allotting the symbols to the faces of the dihexagonal prism and pyramid, it is well to remember the rule that the greater the index, whether positive or negative, the nearer the face is to the corresponding axis in its positive or negative direction as the case may be.

PRACTICAL WORK.

Beryl. c=0.4989. Beryl usually occurs in long prismatic crystals, distinct terminations being exceptional. But the small brilliant crystals from Nerchinsk show the basal

pinakoid, and often various pyramids in addition, such as $\{10\bar{1}1\}$, $\{20\bar{2}1\}$, $\{11\bar{2}1\}$, $\{21\bar{3}1\}$.

Beryl is one of the comparatively few minerals showing the complete symmetry of this class.

Nepheline. c=0.8389. The hexagonal prisms $\{10\bar{1}0\}$ of nepheline, terminated by $\{0001\}$ and sometimes modified by $\{10\bar{1}1\}$, appear to have the full symmetry of the beryl class; but when etched with hydrofluoric acid the faces show asymmetric etching figures, indicating the presence of a hexagonal axis and no other symmetry. This is the symmetry of the hexagonal uniterminal class, VI Mu (see p. 127).

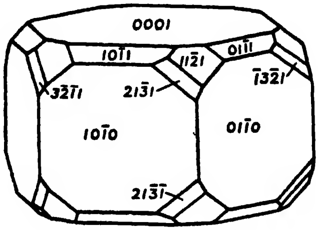


FIG. 133.—Apatite.

Apatite. c=0.7346. This mineral also shows hexagonal prisms $\{10\bar{1}0\}$, terminated by $\{0001\}$ and $\{10\bar{1}1\}$, as in *Fig.* 132, and apparently with the full symmetry of the beryl class. But the occasional presence of twelve small faces (instead of twenty-four) of the form $\{21\bar{3}1\}$, as in *Fig.* 133, shows the absence of the vertical planes and horizontal lines of symmetry and the presence of only a centre, a horizontal plane, and a vertical axis of hexagonal symmetry, which is the symmetry of the hexagonal central class, VI Mc (see p. 127).

CHAPTER XIII.

HEXAGONAL SYSTEM (CONTINUED).

TRIGONAL CLASSES.

CALCITE CLASS

(III Dc, DITRIGONAL CENTRAL).

In the calcite class the principal axis is one of trigonal symmetry, iii., there are three horizontal digonal axes, ii., a centre of symmetry and three vertical planes of symmetry (*Fig.* 134). The lines of symmetry are taken as lateral crystallographic axes, and the planes of symmetry are per-

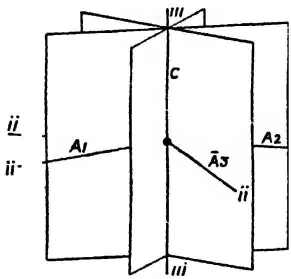


FIG. 134.—Calcite Class—Symmetry, c, 3P, iii, 3 ii.

pendicular to these lines, and therefore parallel to the planes $11\bar{2}0$, $1\bar{2}10$ and $\bar{2}110$.

The basal pinakoid {0001} consists of two faces as in the beryl class, since a centre of symmetry requires two opposite parallel faces.

The prisms of the first and second orders are also unchanged. The axis of one-third-turn symmetry requires three similar faces, and the centre of symmetry provides

that each of these shall be accompanied by an opposite parallel face.

The dihexagonal prism also is the same as in the beryl class, the vertical planes and centre of symmetry (or the half-turn-axes) requiring the full twelve faces.

The face $10\bar{1}1$ must be accompanied by the faces $\bar{1}101$ and $0\bar{1}11$ to satisfy the trigonal axis or the three planes of symmetry. The centre of symmetry and the three digonal axes are satisfied if to these are added their opposite faces, $\bar{1}01\bar{1}$, $1\bar{1}0\bar{1}$ and $01\bar{1}\bar{1}$. The form $\{10\bar{1}1\}$ therefore consists of six faces only (*Fig.* 135).

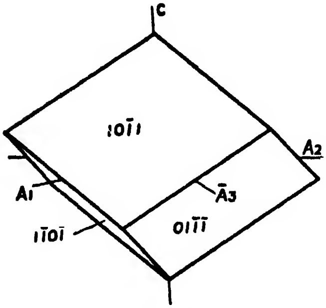


Fig. 135.—Positive Rhombohedron.

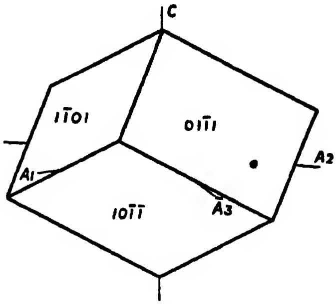


Fig. 136.—Negative Rhombohedron.

The faces are rhombs and the form is known as a r h o m b o h e d r o n. The first three indices of the upper faces are in descending cyclic order (1, 0, $\bar{1}$) and those of the lower faces in ascending ($\bar{1}$, 0, 1). The reverse is the case in the similar form $\{01\bar{1}1\}$, shown in *Fig.* 136, which is often called a negative rhombohedron, while $\{10\bar{1}1\}$ is a positive rhombohedron.*

Rhombohedra may be more acute than the unit form, such as $\{40\bar{4}1\}$, or more obtuse, as $\{01\bar{1}2\}$.

In its general form the rhombohedron has the following faces :—

* A negative rhombohedron is referred to by Professor Lewis as a direct rhombohedron, and a positive rhombohedron as an inverse rhombohedron.

Positive rhombohedron.

$$h0\bar{h}l,\quad \bar{h}h0l,\quad 0\bar{h}hl,$$
$$\bar{h}0h\bar{l},\quad h\bar{h}0\bar{l},\quad 0h\bar{h}\bar{l}.$$

Negative rhombohedron.

$$0h\bar{h}l,\quad \bar{h}0hl,\quad h\bar{h}0l,$$
$$0\bar{h}h\bar{l},\quad h0\bar{h}\bar{l},\quad \bar{h}h0\bar{l}.$$

The face $11\bar{2}1$ is inclined to two of the planes of symmetry, which require the corresponding faces $2\bar{1}\bar{1}1$ and $\bar{1}2\bar{1}1$. The third plane of symmetry repeats these faces in the corresponding faces $\bar{1}\bar{1}21$, $1\bar{2}11$ and $\bar{2}111$, and the centre of symmetry requires that the six faces opposite and parallel to these shall also be present. The

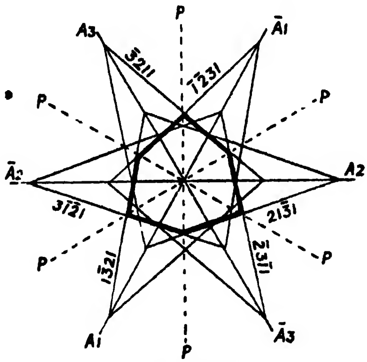


FIG. 137.—The Upper Faces of a Scalenohedron projected on a horizontal plane.

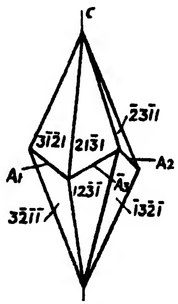


FIG. 138.—Scalenohedron.

form $\{11\bar{2}1\}$, or $\{hh\overline{2h}l\}$, is therefore an hexagonal pyramid of the second order, as in the beryl class.

The face $21\bar{3}1$ is inclined to the front and rear plane of symmetry, a broken line PP in *Fig.* 137, and must therefore be accompanied by $3\bar{1}\bar{2}1$. The other two planes of symmetry PP repeat this pair of faces in alternate sextants*, and the six faces shown in *Fig.* 137 satisfy these planes

* By a sextant is meant one of the six spaces included between adjoining lateral crystallographic axes and meeting in the vertical axis.

and the trigonal axis. The centre or the axes of half-turn symmetry require six faces opposite to these, that is, having their like edges in the three sextants not occupied by such edges in the upper part of the crystal. Thus a sharp edge in the lower part will come below a blunt edge in the upper, and *vice versa*. The result (*Fig.* 138) is a form with twelve faces which are scalene triangles. Such a form is called a s c a l e n o h e d r o n. The six lateral edges of this scalenohedron $\{21\bar{3}1\}$ are the same as those of the positive rhombohedron $\{10\bar{1}1\}$; it is therefore called a positive scalenohedron. The form $\{12\bar{3}1\}$ is a negative scalenohedron, with the lateral edges coinciding with those of the negative rhombohedron $\{01\bar{1}1\}$. The general form of the positive scalenohedron is $\{h_2\, h_1\, \bar{h}_3\, l\}$ and the negative scalenohedron $\{h_1\, h_2\, \bar{h}_3\, l\}$, where h_1 is less than h_2 and $h_3 = h_1 + h_2$. The faces present in these two forms are :—

Positive scalenohedron.

$$\begin{array}{lll}
h_2\, h_1\, \bar{h}_3\, l, & \bar{h}_2\, h_3\, \bar{h}_1\, l, & \bar{h}_3\, h_2\, h_1\, l, \\
\bar{h}_1\, \bar{h}_2\, h_3\, l, & h_1\, \bar{h}_3\, h_2\, l, & h_3\, \bar{h}_1\, \bar{h}_2\, l, \\
\bar{h}_2\, \bar{h}_1\, h_3\, \bar{l}, & h_2\, \bar{h}_3\, h_1\, \bar{l}, & h_3\, \bar{h}_2\, \bar{h}_1\, \bar{l}, \\
h_1\, h_2\, \bar{h}_3\, \bar{l}, & \bar{h}_1\, h_3\, \bar{h}_2\, \bar{l}, & \bar{h}_3\, h_1\, h_2\, \bar{l}.
\end{array}$$

Negative scalenohedron.

$$\begin{array}{lll}
h_1\, h_2\, \bar{h}_3\, l, & \bar{h}_1\, h_3\, \bar{h}_2\, l, & \bar{h}_3\, h_1\, h_2\, l, \\
\bar{h}_2\, \bar{h}_1\, h_3\, l, & h_2\, \bar{h}_3\, h_1\, l, & h_3\, \bar{h}_2\, \bar{h}_1\, l, \\
\bar{h}_1\, \bar{h}_2\, h_3\, \bar{l}, & h_1\, \bar{h}_3\, h_2\, \bar{l}, & h_3\, \bar{h}_1\, \bar{h}_2\, \bar{l}, \\
h_2\, h_1\, \bar{h}_3\, \bar{l}, & \bar{h}_2\, h_3\, \bar{h}_1\, \bar{l}, & \bar{h}_3\, h_2\, h_1\, \bar{l}.
\end{array}$$

In drawing rhombohedra and scalenohedra the ground plan (as seen from above) should first be drawn. This view is bounded by a regular hexagon in each case, and it gives the vertical construction lines for the front and side views. The points of a rhombohedron lie on four equidistant horizontal planes, represented by horizontal lines in the front and side views; for a scalenohedron the two inner planes are usually closer to each other than either of them is to the top or bottom plane. In some crystals, however,

they are farther apart. *Fig.* 139 represents the cleavage rhombohedron of calcite, and *Fig.* 140 the scalenohedron most commonly seen in that mineral.

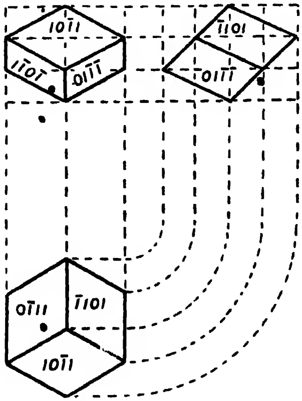


FIG. 139.—Rhombohedron.

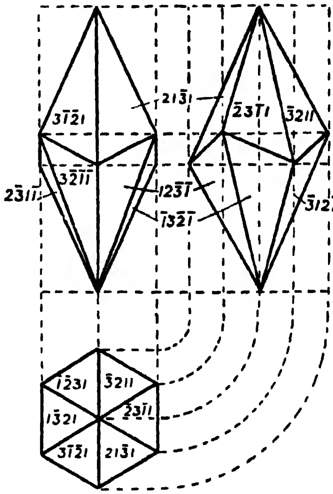


FIG. 140.—Scalenohedron.

TOURMALINE CLASS

(III Du, DITRIGONAL UNITERMINAL).

This class has a vertical axis of one-third-turn symmetry and three vertical planes of symmetry, but no centre or half-turn axes of symmetry (*Fig.* 141). As a consequence faces occurring at one end of the principal axis are not repeated at the other, and crystals of this class are uniterminal. The crystallographic axes are placed at right angles to the planes of symmetry, as in the calcite class, but they are not axes of symmetry.

The basal pinakoid is reduced to one face only, 0001 or 000$\bar{1}$, since there is no centre or horizontal line of symmetry to necessitate the existence of both faces if one is present.

The prism face $10\bar{1}0$ must be accompanied by $\bar{1}100$ and $0\bar{1}10$. These three faces satisfy the trigonal axis and the three planes of symmetry, and therefore the form $\{10\bar{1}0\}$ consists of these faces only. It is called a *trigonal prism of the first order*, and is shown in *Fig.* 142. The form $\{01\bar{1}0\}$ is an independent trigonal prism of the first order.

The prism face $11\bar{2}0$ is parallel to a plane of symmetry. Consequently the opposite face $\bar{1}\bar{1}20$ must also be present. The trigonal axis will add two more faces to

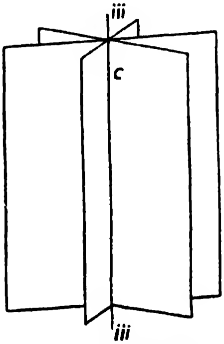


FIG. 141.—Tourmaline Class—Symmetry, 3P, iii.

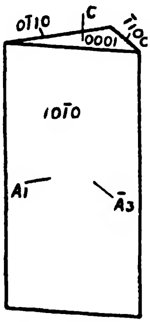


FIG. 142.—Trigonal Prism of the First Order.

each of these, making six in all. The form $\{11\bar{2}0\}$ therefore is the hexagonal prism of the second order, as in the beryl class.

The prism face $12\bar{3}0$ must be accompanied by $\bar{1}3\bar{2}0$, as they adjoin each other on opposite sides of a plane of symmetry, and these two faces are repeated by the threefold axis in alternate sextants. The form $\{12\bar{3}0\}$ is therefore a six-faced prism, the edges of which are sharper and blunter alternately. It is known as a *ditrigonal prism* (*Fig.* 143), and the general form $\{h_1 h_2 \bar{h}_3 0\}$, where $h_3 = h_1 + h_2$, has the faces

$$h_1 h_2 \bar{h}_3 0, \quad \bar{h}_1 h_3 \bar{h}_2 0, \quad \bar{h}_3 h_1 h_2 0,$$
$$\bar{h}_2 \bar{h}_1 h_3 0, \quad h_2 \bar{h}_3 h_1 0, \quad h_3 \bar{h}_2 \bar{h}_1 0.$$

The trigonal axis requires that the face $10\bar{1}1$ must be accompanied by $\bar{1}101$ and $0\bar{1}11$, but these three faces satisfy all the symmetry of the class. The form $\{10\bar{1}1\}$, or $\{h0\bar{h}l\}$, therefore consists of the three upper faces of a rhombohedron, the lower faces of which would constitute an entirely independent form, $\{01\bar{1}\bar{1}\}$ or $\{0h\bar{h}\bar{l}\}$.

Similarly the form $\{11\bar{2}1\}$, or $\{hh\overline{2h}l\}$, consists of the six upper faces of an hexagonal pyramid of the second order, and $\{12\bar{3}1\}$, or $\{h_1 h_2 \bar{h}_3 l\}$, consists of the six upper faces of a scalenohedron.

Most of these forms will be seen in *Fig.* 144, which represents a crystal of tourmaline. The form $\{11\bar{2}0\}$

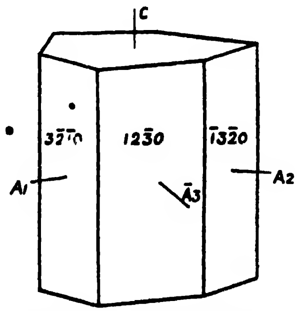


FIG. 143.—Ditrigonal Prism.

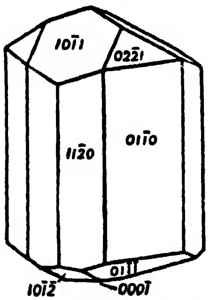


FIG. 144.—Tourmaline.

alone has the same number of faces in the tourmaline class as in the beryl class. The forms $\{0001\}$, $\{10\bar{1}0\}$, $\{h_1 h_2 \bar{h}_3 0\}$ and $\{hh\overline{2h}l\}$ have half the number of faces, and the forms $\{h0\bar{h}l\}$ and $\{h_1 h_2 \bar{h}_3 l\}$ one-fourth the number of faces of the corresponding forms in the beryl class, and are sometimes called hemihedral and tetartohedral forms accordingly (Gr. *hemi*, half, *tetartos*, fourth).

Crystals of the tourmaline class are often called h e m i m o r p h i c, which means the same as uniterminal. Like most hemimorphic minerals tourmaline is strongly pyro-electric, the unlike terminations developing opposite electrical charges during the warming or cooling of the crystal. Usually the more obtuse end of a tourmaline

crystal is the analogous end, *i.e.*, the end at which addition of heat produces a positive and loss of heat a negative charge; the other end is said to be antilogous.

QUARTZ CLASS*

(III Mh, TRIGONAL HOLOAXIAL).

In the quartz class the principal axis is one of one-third-turn symmetry, and the three lateral axes are lines of half-turn symmetry (*Fig.* 145). There are no planes of symmetry and no centre of symmetry.

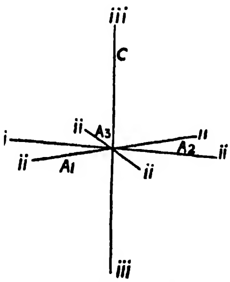


FIG. 145.—Quartz Class—Symmetry, iii, 3 ii.

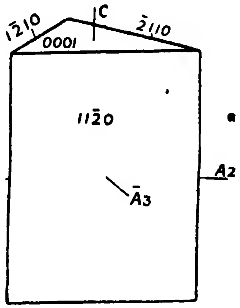


FIG. 146.—Trigonal Prism of the Second Order.

The basal pinakoid consists of the two faces 0001 and $000\bar{1}$, the horizontal lines of symmetry requiring both faces if one is present.

The prism face $10\bar{1}0$ must be accompanied by the opposite face $\bar{1}010$ to satisfy one of the horizontal half-turn axes. The vertical axis of three-fold symmetry repeats $10\bar{1}0$ in $\bar{1}100$ and $0\bar{1}10$, and $\bar{1}010$ in $1\bar{1}00$ and $01\bar{1}0$. The form $\{10\bar{1}0\}$ is therefore the hexagonal prism of the first order, as in the beryl class.

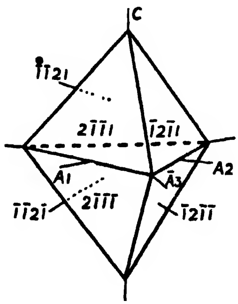
The trigonal axis requires that the prism face $11\bar{2}0$ must be accompanied by $\bar{2}110$ and $1\bar{2}10$, and these three faces satisfy all the symmetry of the class, for the

three digonal axes of symmetry are at right angles to them and require the presence of no other faces. The form $\{11\bar{2}0\}$ is therefore a three-faced prism, and may be called the trigonal prism of the second order (*Fig.* 146). It is rarely seen in quartz.

The prism face $12\bar{3}0$ must be accompanied by five others giving a ditrigonal prism with sharper and blunter edges alternately. The general form of this prism has the faces

$$h_1\,h_2\,\bar{h}_3\,o, \qquad \bar{h}_3\,h_2\,h_1\,o, \qquad \bar{h}_3\,h_1\,h_2\,o,$$
$$h_1\,\bar{h}_3\,h_2\,o, \qquad h_2\,\bar{h}_3\,h_1\,o, \qquad h_2\,h_1\,\bar{h}_3\,o.$$

These are alternate pairs of faces of the dihexagonal prism of the beryl class, but it will be seen that the largest index,



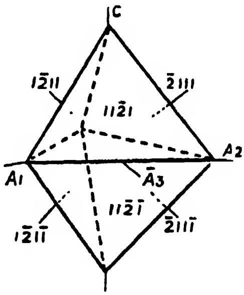


Fig. 147. Fig. 148.

Trigonal Pyramids.

h_3, is of the same sign throughout, whereas in the tourmaline class it is alternately positive and negative in successive faces. This form is very rare in quartz.

The face $10\bar{1}1$ is repeated about the vertical axis in the faces $\bar{1}101$ and $0\bar{1}11$, and the horizontal digonal axes give three corresponding faces below, namely, $01\bar{1}\bar{1}$, $\bar{1}01\bar{1}$ and $1\bar{1}0\bar{1}$. Thus the positive and negative rhombohedra, $\{h\,o\,\bar{h}\,l\}$ and $\{o\,h\,\bar{h}\,l\}$, are the same as in the calcite class.

The face $11\bar{2}1$ must be accompanied by $\bar{2}111$ and $1\bar{2}11$ around the vertical axis. The horizontal

lines of symmetry give the lower faces $11\bar{2}\bar{1}$, $\bar{2}11\bar{1}$ and $1\bar{2}1\bar{1}$, which are not parallel to the upper faces. These six faces, if occurring alone, would form a trigonal pyramid with triangular faces. In the general forms $\{hh\overline{2h}l\}$ and $\{\bar{h}2h\bar{h}l\}$ the greater lateral index is either positive or negative throughout (*Figs.* 147, 148).

The face $21\bar{3}1$ must have $\bar{3}211$ and $1\bar{3}21$ to satisfy the trigonal axis. These faces are repeated below by the horizontal lines of symmetry as $\bar{3}12\bar{1}$, $2\bar{3}1\bar{1}$ and $12\bar{3}\bar{1}$. The form $\{21\bar{3}1\}$ therefore has six faces, one-half the faces of the corresponding scalenohedron or one-quarter

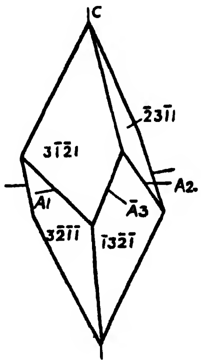


FIG. 149.

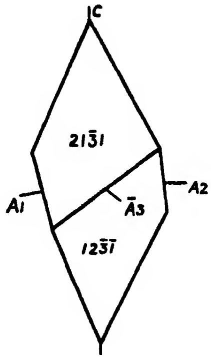


FIG. 150.

Trigonal Trapezohedra.

those of the dihexagonal pyramid. As usually developed in quartz, faces of the general forms $\{h_2 h_1 \bar{h}_3 l\}$ and $\{\bar{h}_2 h_3 \bar{h}_1 l\}$ are trapezoids, and these forms are therefore known as t r i g o n a l t r a p e z o h e d r a (*Figs.* 149, 150).

The only important forms peculiar to the quartz class are $\{hh\overline{2h}l\}$ and $\{h_2 h_1 \bar{h}_3 l\}$, and these only occur in combinations, usually with very small faces. The figures show that each pair of related forms, differing only in the sign of the greatest lateral index, resembles a pair of gloves, in that one form is the mirror-image of the other. Such pairs of

forms are said to be e n a n t i o m o r p h i c (Gr. *enantios*, opposite, *morphe*, form).

Quartz crystals having, usually, the forms $\{2\bar{1}\bar{1}1\}$ and $\{6\bar{1}\bar{5}1\}$ are said to be left-handed, as the faces of these forms are arranged in a manner suggesting a left-handed screw (*Fig.* 151), while others are right-handed, having $\{11\bar{2}1\}$ and $\{51\bar{6}1\}$, which resemble a right-handed screw (*Fig.* 152). A close connection between crystal form and molecular structure is suggested by the fact that left-handed quartz crystals rotate the plane of vibration of polarised light, propagated parallel to the vertical axis, to

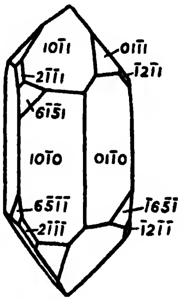


FIG. 151.
Left-handed Quartz.

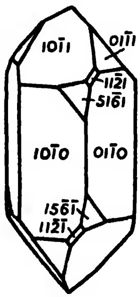


FIG. 152.
Right-handed Quartz.

the left, while right-handed crystals rotate it to the right. In other words, in right-handed crystals the rotation is clockwise if the light is moving from the back of the clock to the front.

The symbol of any of the rhomb faces in quartz is easily obtained by adding the indices of the adjacent rhombohedron and prism faces, whose edge it truncates, thus; $10\bar{1}1 + 01\bar{1}0 = 11\bar{2}1$. The trapezoidal face between this face and the prism face $10\bar{1}0$ is found by adding four times the indices of the latter (since the face lies closer to it) to those of the former, thus $40\bar{4}0 + 11\bar{2}1 = 51\bar{6}1$.

The number of faces present in each form in the various

classes of the hexagonal system that we have mentioned is shown in the following table :—

Beryl Class.	Calcite Class.	Tourmaline Class.	Quartz Class.
	$\{h_1 h_2 \bar{h}_3 l\}$, *e.g.*, $\{21\bar{3}1\}$		
Dihexagonal pyramid, 24.	Scalenohedron, 12.	Hemi-scalenohedron, 6.	Trigonal trapezohedron, 6.
	$\{h h \overline{2h} l\}$, *e.g.*, $\{11\bar{2}1\}$		
Pyramid of the second order, 12.	Pyramid of the second order, 12.	Hemi-pyramid of the second order, 6.	Trigonal pyramid, 6.
	$\{h o \bar{h} l\}$, *e.g.*, $\{10\bar{1}1\}$		
Pyramid of the first order, 12.	Rhombohedron, 6.	Hemi-rhombohedron, 3.	Rhombohedron, 6.
	$\{h_1 h_2 \bar{h}_3 o\}$, *e.g.*, $\{12\bar{3}0\}$		
Dihexagonal prism, 12.	Dihexagonal prism, 12.	Ditrigonal prism, 6.	Ditrigonal prism, 6.
	$\{11\bar{2}0\}$		
Prism of the second order, 6.	Prism of the second order, 6.	Prism of the second order, 6.	Trigonal prism of the second order 3.
	$\{10\bar{1}0\}$		
Prism of the first order, 6.	Prism of the first order, 6.	Trigonal prism of the first order, 3.	Prism of the first order, 6.
	$\{0001\}$		
Basal pinakoid, 2.	Basal pinakoid, 2.	Basal plane, 1.	Basal pinakoid, 2.

PRACTICAL WORK.

Calcite. c=0.8543. Calcite crystals show a great variety of habits — acute and obtuse rhombohedra and scalenohedra, prisms, and thin tabular forms. The commonest forms are the scalenohedron {$21\bar{3}1$} and the cleavage rhombohedron {$10\bar{1}1$} (*Figs.* 138, 135); while a combination of the prism {$10\bar{1}0$} with the rhombohedron {$01\bar{1}2$} is often seen (*Fig.* 153). Crystals are sometimes highly modified.

Dolomite. c=0.8322. This member of the rhombohedral carbonates does not exhibit the same abundance of forms as calcite. It occurs usually as simple rhombohedra,

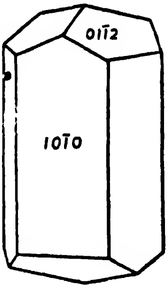


FIG. 153.—Calcite.

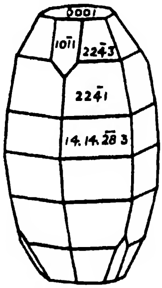


FIG. 154.—Corundum.

{$10\bar{1}1$} or {$40\bar{4}1$}. In some crystals the presence of six faces only of such a form as {$32\bar{5}1$} (a rhombohedron of the third order) indicates the lower symmetry of the phenakite class, with one trigonal axis and a centre of symmetry only. This is the trigonal central class, III Mc (see p. 127).

Corundum. c=1.3630. Perhaps the most characteristic habit is the barrel-shaped crystal made up of second-order pyramids, such as {$22\bar{4}3$}, {$22\bar{4}1$} and {$14.14.\overline{28}.3$}, with {0001} and {$10\bar{1}1$}, as shown in *Fig.* 154.

Hæmatite. c=1.3656. Crystals are usually tabular parallel to 0001, with {$10\bar{1}1$} at the edges.

Ilmenite. c = 1.3846. Resembles hæmatite in habit, but sometimes shows the symmetry of the phenakite class in the presence of rhombohedra of the second and third orders.

Tourmaline. c=0.4477. The habit is usually prismatic, sometimes flattened, the trigonal cross-section and the strong vertical striation due to oscillatory combination of $\{10\bar{1}0\}$ and $\{11\bar{2}0\}$ being characteristic. The hemimorphic character is observable only in complete crystals (*Fig*. 155).

Quartz. c = 1.09997. Many quartz crystals, such as *Fig*. 156, appear to possess the symmetry of the beryl class, with hexagonal prism (horizontally striated) and pyramid.

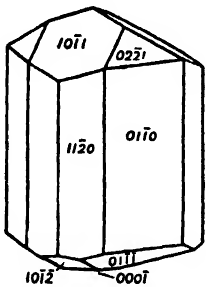


FIG. 155.—Tourmaline.

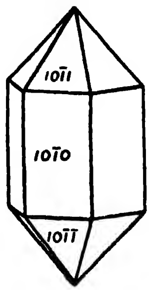


FIG. 156.—Quartz.

The prism is sometimes wanting. In other specimens the two rhombohedra $\{10\bar{1}1\}$ and $\{01\bar{1}1\}$ are unequally developed, simulating calcite-class symmetry. Comparatively rarely are small faces of such forms as $\{11\bar{2}1\}$ and $\{51\bar{6}1\}$ present to show the true symmetry (*Figs*. 151, 152).

Cinnabar is another mineral crystallising in the quartz class.

CHAPTER XIV.

TWIN-CRYSTALS.

A twin-crystal consists of two portions (at least), which have a different orientation, but which are symmetrically related to each other. Both portions consist of the same substance with identical or enantiomorphic structure (see p. 109).

Twinning may be regarded as an unsuccessful attempt to establish a symmetry higher than that to which the simple crystal belongs.

In many cases the structures of the two parts can be made to coincide by a rotation of one of them relatively to the other through a half-turn, about a common axis, thus simulating a half-turn or digonal axis, which is not present. This is line or rotation-twinning.

Frequently, too, coincidence can be obtained by the reflection of one component on a common plane, as if it were a plane of symmetry, which it, in fact, is not in the simple crystal. This is plane or reflection-twinning.

Less commonly one portion may be considered as representing the inversion of the other about a point as if there were a centre of symmetry. Such twinning can, of course, only occur where the untwinned crystal has no centre of symmetry. This is point or inversion-twinning.

If there is a centre of symmetry in the untwinned crystal every twin may be regarded either as the result of the rotation of one component through a half-turn about a line or of a reflection about a plane at right angles to that line. Both processes produce the same result. The line is then termed a twin-axis and the plane a twin-plane.

If there is no centre of symmetry and there is rotation-

twinning about a line, that may be regarded as the twin-axis, and the plane at right angles to it as the twin-plane.

Similarly, if there is only reflection-twinning, the plane of reflection may be considered as the twin-plane and its normal as the twin-axis.

In all these cases the twin-axis and twin-plane and every line in the latter have the same crystallographic position in both crystals.

This is true of any line whatever in an inversion-twin, so that in such a twin any line might be taken as a twin-axis and the plane at right angles to it as a twin-plane.

The components of a twin-crystal are as a rule united in a plane known as the plane of composition. It is usually a twin-plane, but occasionally it is another plane passing

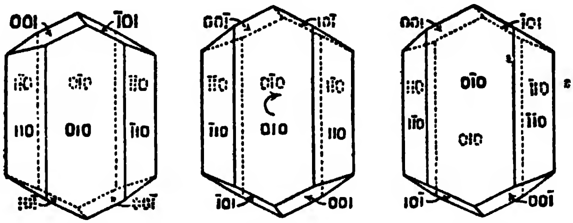


FIG. 157. IMc. FIG. 158. IMc L. FIG. 159. IMc P.

through the twin-axis and having only one or only a limited number of pairs of lines (at right angles to each other), which have the same crystallographic position in both crystals. Such a plane is termed a cross-plane.

Sometimes there is no plane of composition and the two components interpenetrate on an irregular surface.

In a twin-crystal there is usually either a twin-plane parallel to a possible crystal face, or a twin-axis parallel to a possible crystal edge, or both.

If the simple crystal possess symmetry it will usually be possible to explain a twin-crystal by more than one mode of twinning, or by the same mode of twinning with more than one twin-axis.

(1) The commonest and most important case is that

already alluded to in which the untwinned crystal possesses a centre of symmetry. As we have seen, the same result is obtained by reflection on the twin-plane and by rotation on the twin-axis. This is illustrated by *Figs.* 157 to 159, which represent the character of the albite twinning of the triclinic mineral albite, in which the brachypinakoid is the twin-plane. The first, *Fig.* 157, represents the supposed original position of a component of the twin projected on the brachypinakoid or twin-plane. *Fig.* 158 shows the result of a rotation about the twin-axis, which is at right angles to the twin-plane, the brachypinakoid, and the plane of the paper, and *Fig.* 159 that of a reflection on the twin-plane, that is, on the plane of the paper. It will be seen that the second and third figures are identical. In the symbols under these and other twin figures, the numeral and the first two letters express the symmetry of the t w i n - a x i s alone on the principles explained on pp. 124-125 for the symmetry of the principal axis of a crystal, and the last letters, L and P, rotation (line) and reflection (plane) twinning respectively. In the same manner the letter I indicates inversion twinning. The movements which are supposed to have taken place are indicated by the indices of the faces shown. The edges at the back and the indices of the back faces are dotted. It will be seen that the rotated and reflected components have indices of opposite signs. This is always the case.

(2) If there be a line of symmetry (see p. 9) in an untwinned crystal, and if there be an axis of rotation- or of reflection-twinning or of both at right angles to it, then there will be another twin-axis of the same type at right angles both to the first twin and to the line of symmetry.

This is illustrated by a twin of quartz, in which there is a horizontal axis of rotation-twinning at right angles to a digonal axis of symmetry. In such twins there is another axis of rotation-twinning parallel to the vertical axis (see *Figs.* 160 and 161). In the same manner if there is a horizontal axis of reflection-twinning at right angles to a

digonal axis, the vertical axis will also be an axis of reflection-twinning (see *Figs.* 160 and 162). As quartz has no centre of symmetry, these twins will be distinct from one another. A simple line on the margin of a figure indicates the direction of an axis of rotation-twinning, a line with another at right angles to it that of an axis of reflection-twinning.

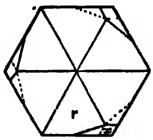


FIG. 160.
IIIMh.

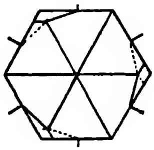


FIG. 161.
IIIMh L.

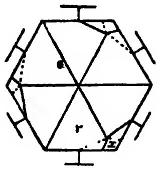


FIG. 162.
IIIMh P.

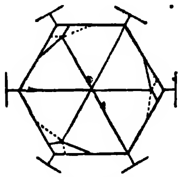


FIG. 163.
IIIMh I.

The case where there is also a centre of symmetry and each twin-axis is accordingly an axis both of rotation- and reflection-twinning, is illustrated by the Carlsbad-twins of orthoclase. Here there are two axes of twinning, one the vertical axis, the other the normal to the orthopinakoid. Both are at right angles to the ortho-axis, which is here an

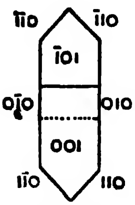


FIG. 164.
IDc.

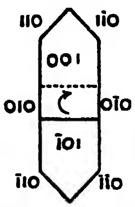


FIG. 165.
(*a*) IDc L.

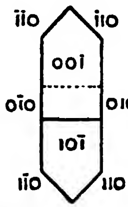


FIG. 166.
(*b*) IDc P.

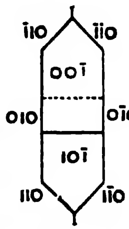


FIG. 167.
(*c*) IDc L.

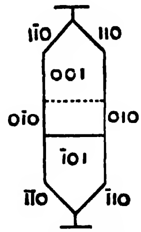


FIG. 168.
(*d*) IDc P.

axis of digonal symmetry and therefore a line of symmetry, and lie in the clinopinakoid, which is a plane of symmetry; and they are at right angles to each other.

Orthographic projections of a crystal of orthoclase on the plane at right angles to the vertical axis are shown in *Figs.* 164 to 168. They demonstrate that the same configuration may be brought about by different operations,

which can be inferred from the indices of the faces shown. *Fig.* 165 shows the positions of the faces after (*a*) rotation on the vertical axis; *Fig.* 166 after (*b*) reflection on the horizontal plane to which it is normal; *Fig.* 167 after (*c*) rotation on the normal to the orthopinakoid; and *Fig.* 168 after (*d*) reflection on the orthopinakoid. All these twinning-operations produce an indistinguishable result.

Figs. 169 to 173 are orthographic projections on the clinopinakoid corresponding to *Figs.* 164 to 168.

If (as in plagioclase) there were no symmetry except the centre of symmetry, there could be only one twin-axis in the same simple twin, corresponding either to (*a*) and (*b*) or to (*c*) and (*d*). There are in fact two different kinds of twins in triclinic felspars corresponding to the Carlsbad-twin of orthoclase. In plagioclase the twin-axis is usually the

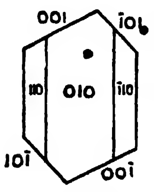


FIG. 169. IIMc.

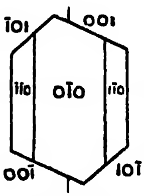


FIG. 170. (*a*) IIMc L.

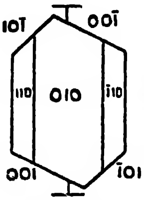


FIG. 171. (*b*) IIMc P.

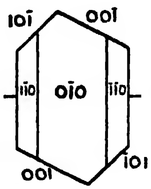


FIG. 172. (*c*) IIMc L.

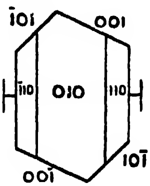


FIG. 173. (*d*) IIMc P.

vertical axis, and in anorthoclase, apparently, the normal to the macropinakoid.

If the centre of symmetry were absent and only the axis of digonal symmetry remained, the two twin-axes would both be either axes of rotation-twinning (*a*) and (*c*), or both axes of reflection-twinning (*b*) and (*d*), but not axes of both kinds of twinning. We shall see later that if only the plane of symmetry remained, one of the twin-axes would be an axis of rotation-twinning and the other of reflection-twinning, so that (*a*) and (*d*) would be equivalent twinning-operations and so would (*b*) and (*c*), but the two former would not give the same result as the two latter.

(3) If a line of symmetry be an axis of reflection-twinning, the twin-crystal can also be explained by inversion-

twinning, and, conversely, if a twin-crystal can be explained by inversion-twinning every line of symmetry is an axis of reflection-twinning.

There are twins of eulytine, diamond, and tetrahedrite, which can be described as twins by reflection on the cube faces. The crystallographic axes are therefore axes of reflection-twinning. As they are at the same time lines of symmetry, being digonal axes, the twins may also be explained by inversion-twinning, so that every point of one component portion corresponds to an exactly similar point of the other on the opposite side of the point of inversion. See *Figs.* 174 and 175.

If one of the digonal axes of quartz be an axis of reflection-twinning, the twinning may also be regarded as an inversion-twin. Thus *Fig.* 163 is an inversion of *Fig.* 160.

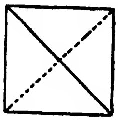


FIG. 174.
IVDv.

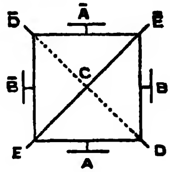


FIG. 175.
IVDv PI.

A line of symmetry (such as the digonal axes of quartz) cannot be an axis of rotation-twinning.

A plane of symmetry will also give rise to equivalent twin-operations.

(4) If there be an axis of rotation-twinning parallel to a plane of symmetry, there will be parallel to the same plane of symmetry an axis of reflection-twinning at right angles to the axis of rotation-twinning; and, *vice versa*, an axis of reflection-twinning parallel to a plane of symmetry implies the existence of an axis of rotation-twinning at right angles to it and parallel to the same plane of symmetry. Examples may occur in any crystal possessing a plane of symmetry but no centre of symmetry.

If there be an axis of both rotation- and reflection-twinning parallel to a plane of symmetry, there will be another

axis of the same kind of twinning parallel to the same plane of symmetry and at right angles to the first axis. This case is, however, the same as the case of a similar axis of twinning at right angles to a line of symmetry, for a centre of symmetry must be present (see pp. 113, 115), and when that is the case every plane of symmetry has a line of symmetry at right angles to it. It has already been illustrated by the projections of Carlsbad twin-crystals in *Figs.* 164 to 173. If the centre of symmetry and line of symmetry did not exist, but only the plane of symmetry, each axis would, as stated on p. 117, be an axis of only one mode of twinning, and while one would be an axis of rotation-twinning, the other would be an axis of reflection-twinning.

(5) If the normal to a plane of symmetry be an axis of rotation-twinning, the same twin-crystal can be formed by inversion-twinning; and if a twin-crystal can be explained by inversion-twinning, the normal to every plane of symmetry is an axis of rotation-twinning. Thus the twins of eulytine, diamond, and tetrahedrite, already referred to (p. 118, and *Figs.* 174 and 175), possess axes of rotation-twinning normal to the six planes of symmetry parallel to the rhombic-dodecahedron faces.

The normal to a plane of symmetry cannot be an axis of reflection-twinning. The similarity of the relations between equivalent twinning-operations due to the presence of a line or a plane of symmetry, may be seen at a glance in the following table, which also shows how closely the normal to a plane of symmetry is comparable to a line of symmetry.

Line of Symmetry.	*The Normal to a Plane of Symmetry.*
If there be an axis of *rotation-*, or of *reflection-twinning*, or of *both*, at right angles to a line of symmetry, there will be another axis of *rotation-*, or *reflection-twinning*, respectively, or of *both*, at right angles to the line of	If there be an axis of *rotation-*, or of *reflection-twinning*, or of *both*, at right angles to the normal to a plane of symmetry, there will be an axis of *reflection-*, or *rotation-twinning* respectively, or of *both*, at right angles

Line of Symmetry.	*The Normal to a Plane of Symmetry*
symmetry and to the first axis of twinning.	to the normal to the plane of symmetry and to the first axis of twinning.
	These twin-axes will, of course, be parallel to the plane of symmetry.
If a line of symmetry be an axis of reflection twinning, the twin-crystal is an inversion-twin.	If the normal to a plane of symmetry be an axis of rotation-twinning, the twin-crystal is an inversion-twin.
In an inversion-twin, every line of symmetry is an axis of reflection-twinning.	In an inversion-twin, every normal to a plane of symmetry is an axis of rotation-twinning.

A plane of symmetry cannot be a plane of reflection-twinning.

Axes of symmetry also give rise in many cases to the multiplication of twin-axes at right angles to them, but it is unnecessary to continue the subject further here. Full information on the subject will be found in a paper in the Mineralogical Magazine, Vol. xviii., 1918, pp. 224-243.

Sometimes the same twin-crystals are composed of a number of differently orientated portions. If these are connected by repetitions of the same mode of twinning on the same axis, as in the albite twins of plagioclase, they form thin plates or lamellæ. This is termed lamellar twinning. In other cases of multiple twinning there are different modes of twinning, or twinning of the same mode on different axes, for instance, the albite and Carlsbad twins of plagioclase.

When an orthorhombic crystal is twinned twice in succession on faces of the same prism or dome, which make an angle of about 60° with each other, the result is a combination which has approximately trigonal symmetry, and has been described as a t r i p l e t. This is well illustrated by the twinning of aragonite.

PRACTICAL WORK.

A series of models of twinned crystals should be drawn. They should be drawn in such a manner that the twin-plane

is at right angles to the plane of the paper, and the vertical axis of one of the components is vertical and in the plane of the paper. The indices of this component may be marked as usual. The indices of the other component should be marked twice over, the first set indicating the faces that

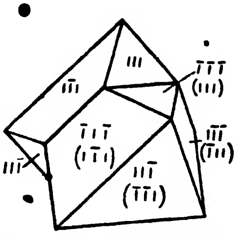


FIG. 176.—Spinel, twinned on $11\bar{1}$.

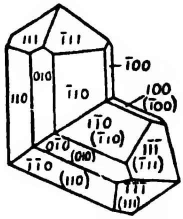


FIG. 177.—Rutile, geniculate twin on 101, side view.

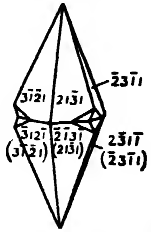


FIG. 178.—Calcite, twinned on 0001.

would be brought into the positions shown after a rotation through 180° about the twin-axis, and the second set those that would be brought there after reflection in the twin-plane. The two sets may be distinguished by the use of ink of different colours, or of parentheses, as in *Figs.* 176 to 181.

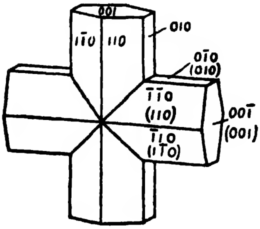


FIG. 179.—Staurolite, twinned on $p3\bar{2}$.

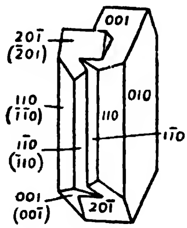


FIG. 180.—Orthoclase, Carlsbad-twin on 100.

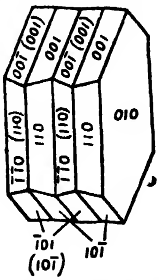


FIG. 181.—Plagioclase, lamellar albite-twinning on 010.

A list of the most important twins is given below. All should be drawn. It must be understood that where twinning is stated to be on a particular face of a form, twinning on any other face of the same form will give an exactly similar result, and in some of the drawings of twins in *Figs.*

176 to 181 it has been found convenient to show the twinning on a face other than that indicated in the list, though belonging to the same form.

In *Fig.* 180 the indices of faces in the twinned component are those required if 100 is the twin-plane and the twin-axis is horizontal. A horizontal twin-plane and a vertical twin-axis would yield indistinguishable results, but different faces would be represented, as explained on pp. 116 and 117.

	Twinning plane.	Plane of composition, if any.
Cubic System.		
Spinel (*Fig.* 176)	111	Twinning plane.
Fluor	111	Interpenetrant.
Tetragonal System.		
Rutile, geniculate twin (*Fig.* 177)	101	Twinning Plane.
Rutile, cordate twin	301	" "
Hexagonal System.		
Calcite, twinned on basal plane (*Fig.* 178)	0001	" "
Calcite, twinned on gliding plane	$01\bar{1}2$	" "
Orthorhombic System.		
Staurolite, cruciform twin (*Fig.* 179)	032	" "
Staurolite	232	" "
Aragonite, pseudo-hexagonal triplets	110	" "
Monoclinic System.		
Gypsum	100	" "
Hornblende	100	" "
Augite	100	" "
Orthoclase, Carlsbad-twin (*Fig.* 180)	100	Interpenetrant, or 010.
Orthoclase, Baveno-twin	021	Twinning plane.
Orthoclase, Manebach-twin	001	" "
Triclinic System.		
Plagioclase, albite-twin (*Fig.* 181)	010	
Kyanite	100	

CHAPTER XV.

THE THIRTY-TWO CLASSES OF CRYSTAL SYMMETRY.

If, as is now generally recognised to be the fact, a typical crystal is made up of innumerable identical cells arranged in an orderly structure of rows and nets, it can be demonstrated that the internal and external symmetry that it exhibits must belong to one of thirty-two different types, usually referred to as the classes of crystal symmetry.

Some of these classes involve a form of axial symmetry which has not been described in earlier chapters. The axes that show it have been termed rotation-inversion or contra-directional axes, as opposed to the ordinary simple or co-directional axes, but for the sake of brevity they will be referred to here as *inverse axes*. Where an inverse axis of symmetry exists in a crystal, an inversion (that is to say the reversal of all directions in the crystal) followed by a rotation through a half, a quarter, or one-sixth of a turn (as the case may be) about the inverse axis will leave the original configuration of the crystal unchanged. According as the minimum rotation required to produce this result is a half, quarter or one-sixth turn, the axis is described as a *digonal*, *tetragonal*, or *hexagonal inverse* axis. There can be no trigonal inverse axes.

It is easily seen that a digonal inverse axis is equivalent to a plane of symmetry at right angles to it; that an hexagonal inverse axis is equivalent to a trigonal axis, combined with a plane of symmetry at right angles to it; and that a tetragonal inverse axis necessarily implies the existence of a digonal simple axis. The only one of these inverse axes that is of practical importance is the tetragonal inverse axis that

occurs in chalcopyrite (copper pyrites). Where nothing is stated as to the character of an axis, a simple axis is of course to be understood.

In the accompanying table the classes are represented by symbols consisting of three parts. The first is a capital Roman numeral indicating the cyclic number of the principal axis, that is to say of the axis or axes of symmetry (whether simple or inverse) with the highest symmetry, II for a digonal axis, IV for a tetragonal axis, VI for an hexagonal axis, and III for a trigonal axis. In the monoclinic system the ortho-axis is the principal axis. In those orthorhombic classes, in which there are three digonal simple axes at right angles to one another, it is a matter of indifference which of the three digonal axes is taken as the principal axis. The numeral I signifies that there is no axial symmetry, and that nothing less than a complete rotation about any axis will bring the crystal into coincidence with its original configuration, so that any zone-axis may be regarded as the principal axis. The five tetratrigonal or cubic classes characterised by the possession of four trigonal axes may be indicated by 4III or by a capital C, the initial letter of cubic. It is true that two of the cubic classes have three tetragonal simple axes and one has three tetragonal inverse axes, but the four trigonal axes are common to all five classes.

The second portion of the class-symbol is a capital letter, either M for monocyclic† or D for dicyclic. In monocyclic classes the principal axis does not lie in a plane of symmetry. Consequently the succession of faces round it is different in opposite directions. In the dicyclic classes the principal axis has two or more planes of symmetry

* For further information with regard to inverse axes, reference may be made to the following publications: H. Hilton, *Min. Mag.*, vol. xiv., pp. 261-263 (1907); J. W. Evans, *Min. Mag.*, vol. xv., pp. 398-400 (1910); *Proc. R. S. Ed.*, vol. xxxii., pp. 418-423 (1912); *Zeitschr. f. Kryst.*, vol. lii., pp. 332-336 (1913).

† In former publications the capital letter U was employed for the monocyclic or "unilateral" classes, and B for the dicyclic or "bilateral" classes. *Min. Mag.*, vol. xv., p. 400 (1910).

passing through it, and the succession of faces round it in opposite directions is consequently the same.

The third portion of the class symbol is a small letter which indicates in what manner the faces of opposite terminations of the principal axis are related. If those at one end are independent of those at the other, the crystal is said to be u n i t e r m i n a l, u, but if the faces at one end are repeated at the other it is b i t e r m i n a l, and the crystals are either c e n t r a l, c, when the faces at opposite ends are connected by a centre of symmetry; i n v e r s e, v,* when their relation is implied by the inverse character of the axis; h o l o a x i a l or h e l i c a l, h, when they are related by two or more digonal axes at right angles to the principal axis.

The names of the classes are based on the crystallographic system to which they belong; triclinic, monoclinic, orthorhombic, tetragonal, hexagonal, trigonal, and tetratrigonal or cubic. Where the name of the system expresses the cyclic number of the principal axis, the syllable di- is prefixed in the dicyclic classes, in accordance with common usage. In the case of the cubic system it is inserted between "tetra" and "trigonal." The monocyclic classes have no corresponding prefix. Then follows one of the terms already explained—uniterminal, central, inverse, or holoaxial, which express the relations between the terminations of the principal axis.

It is, however, frequently convenient to describe a class by referring to a well-known substance that crystallises in it. This course has been followed in the preceding chapters.

In the table the classes have been arranged to bring out clearly the relations between them.

The particulars relating to each class are given in the following order:—

(1) The symbol of the class.

(2) Its name.

(3) The name of a substance (if any) crystallising in it.

* The letter k was formerly employed instead of v.

Distinctive axial symmetry in the different classes. Symbol expressing the cyclic number, k, of such symmetry.	Classes without axial symmetry. I	Classes with digonal symmetry only. II
Names of Systems.	TRICLINIC.	MONOCLINIC AND ORTHORHOMBIC.
MONOCYCLIC, M. No planes of symmetry intersecting		
		MONOCLINIC.
UNITERMINAL, Mu. Terminations of the principal axes of symmetry unlike (hemimorphic). $n=k$.	I Mu *Triclinic uniterminal (asymmetric).* CALCIUM THIOSULPHATE. No symmetry. (1)	II Mu *Monoclinic uniterminal.* CANE SUGAR. ii. axis. (2)
CENTRAL, Mc. Centre of symmetry. $n=2k$.	I Mc *Triclinic central.* ALBITE. Centre. (2)	II Mc *Monoclinic central.* AUGITE. ii. axis + plane of symmetry + centre. (4)
INVERSE, Mv. Terminations of the principal axes of symmetry connected by their inverse (contra-directional) character. $n=k$		II Mv *Monoclinic inverse.* CLINOHEDRITE. Inverse ii. axis = plane of symmetry perpendicular to axis. (2)
HOLOAXIAL or HELICAL, Mh. Terminations of the principal axes of symmetry connected by digonal axes at right angles to them. $n=2k$		ORTHORHOMBIC II Mh *Orthorhombic holoaxial.* EPSOMITE. 3 ii. axes. (4)
DICYCLIC, D. Two or more planes of symmetry		
UNITERMINAL, Du. (hemimorphic). $n=2k$.		II Du *Orthorhombic uniterminal.* HEMIMORPHITE. ii. axis + 2 planes of symmetry. (4)
CENTRAL, Dc. (holohedral). $n=4k$.		II Dc *Orthorhombic central* OLIVINE. 3 ii. axes + 3 planes of symmetry + centre. (8)
INVERSE, Dv. $n=2k$.		(Identical with II Du.)

Classes with one axis of tetragonal symmetry. IV	Classes with an axis of hexagonal symmetry. VI	Classes with one axis of trigonal symmetry. III	Classes with four axes of trigonal symmetry. 4×III or C
TETRAGONAL.	HEXAGONAL.	TRIGONAL.	TETRA-TRIGONAL OR CUBIC.

in the principal axes of symmetry.

IV Mu *Tetragonal uniterminal.* Wulfenite. iv. axis. (4)	VI Mu *Hexagonal uniterminal.* Nepheline. vi. axis. (6)	III Mu *Trigonal uniterminal.* Sodium Periodate. iii. axis. (3)	4 III Mu or C Mu *Tetra-trigonal uniterminal.* Ullmannite. 4 iii. axes + 3 ii. axes. (12)
IV Mc *Tetragonal central.* Scheelite. iv. axis+plane of symmetry+centre. (8)	VI Mc *Hexagonal central.* Apatite. vi. axis+plane of symmetry+centre. (12)	III Mc *Trigonal central.* Phenakite. iii. axis+centre. (6)	4 III Mc or C Mc *Tetra-trigonal central.* Pyrite. 4 iii. axes+3 ii. axes+3 axial planes of symmetry+centre. (24)
IV Mv *Tetragonal inverse.* No example known. Inverse iv. axis =ii. axis. (4)	VI Mv *Hexagonal inverse.* No example known. Inverse vi. axis = iii. axis with plane perpendicular to it. (6)	There are no *odd* inverse axes. Though VI Mu is apparently trigonal, it is really hexagonal.	
IV Mh *Tetragonal holoaxial.* Strychnine Sulphate. iv. axis+4 ii. axes. (8)	VI Mh *Hexagonal holoaxial.* Quartz above 570° C. vi. axis+6 ii. axes. (12)	III Mh *Trigonal holoaxial.* Quartz below 570° C. iii. axis+3 ii. axis. (6)	4 III Mh or C Mh *Tetra-trigonal holoaxial.* Cuprite. 4 iii. axes+3 iv. axes +6 ii. axes. (24)

intersecting in the principal axes of symmetry.

IV Du *Ditetragonal uniterminal.* Succin-iodimide. iv. axis+4 planes of symmetry. (8)	VI Du *Dihexagonal uniterminal.* Iodyrite. vi. axis+6 planes of symmetry. (12)	III Du *Ditrigonal uniterminal.* Tourmaline. iii. axis + 3 planes of symmetry. (6)	4 III Du or C Du *Tetra-ditrigonal uniterminal.* Tetrahedrite. 4 iii. axes+3 inverse iv. axes=3 ii. axes+6 diagonal planes of symmetry. (24)
IV Dc *Ditetragonal central.* Zircon. iv. axis+4 ii. axes+5 planes of symmetry +centre. (16)	VI Dc *Dihexagonal central.* Beryl. vi. axis+6 ii. axes+7 planes of symmetry +centre. (24)	III Dc *Ditrigonal central.* Calcite. iii. axis+3 ii. axes+3 planes of symmetry +centre. (12)	4 III Dc or C Dc *Tetra-ditrigonal central* Spinel. 4 iii. axes+3 iv. axes +6 ii. axes+3 axial and 6 diagonal planes of symmetry+centre. (48)
IV Dv *Ditetragonal inverse.* Chalcopyrite. Inverse iv. axis=ii. axis+2 other ii. axes + 2 planes of symmetry bisecting the angles between the latter. (8)	VI Dv *Dihexagonal inverse.* Benitoite. Inverse vi. axis=iii. axis with plane of symmetry perpendicular to it + 3 ii. axes+3 other planes of symmetry passing through the inverse axis and the ii. axes. (12)	There are no *odd* inverse axes. Though VI Du is apparently trigonal, it is really hexagonal.	

(4) The symmetry of the class.

(5) The number (in parentheses) of faces in the general form, the "symmetry number" of Shearer.

In describing the symmetry the cyclic number of an axis of symmetry is indicated by small Roman numerals, ii, iv, vi or iii. If it is an inverse axis, this is stated. An Arabic numeral, placed before the numeral expressing an axis of symmetry, shows the number of such axes.

All classes the principal axes of which have the same cyclic number are placed in the same column, whether the principal axis be simple or inverse. The last column contains the classes of the tetra-trigonal or cubic system, the distinctive feature of which is, as has been stated, the presence of four trigonal axes.

All the classes of the same system are found in the same column, so that column and system coincide except in the case of the digonal column, which contains three monoclinic and three orthorhombic classes.

The symmetry number, n, is either equal to the cyclic number k, or twice or four times its amount. For this purpose, the cyclic number of the cubic system must be taken as $4 \times \text{iii} = 12$.

The classes in the same horizontal row also show close similarity in their symmetry.

Those in the monocyclic uniterminal, Mu, row have no symmetry other than the axial symmetry common to the whole column to which they belong. The only exception is the tetra-trigonal uniterminal class in which the four trigonal axes are connected by three digonal axes* at right angles to one another. In the Mu row $n=k$.

In the classes of the monocyclic central, Mc, row the central symmetry is added to that of the principal axis, with the result that where the axis is a line of symmetry, that is to say has a cyclic number 2, 4, or 6, there is a plane of symmetry at right angles to it. This is not the case where the cyclic number is 3; but in the tetra-trigonal central class (that of pyrite) there is a plane of symmetry at right angles to each of the three digonal axes. In the Mc row $n=2k$.

* These are not tetragonal inverse axes.

The monocyclic inverse, Mv, row is of course only represented in the three columns with even cyclic numbers. Reference has already been made to the symmetry implied by these axes. In this row $n=k$.

The next is the monocyclic holoaxial, Mh, row. Here there are as many digonal axes at right angles to the principal axis as the cyclic number of the latter. Such a crystal is said to be holoaxial, because it has as many axes of symmetry as any class in the same column, but has no other symmetry.* In the tetra-trigonal holoaxial class there are six digonal axes, and each of the four trigonal axes has three of these digonal axes at right angles to it. In the Mh row $n=2k$.

The dicyclic uniterminal, Du, row has as many planes of symmetry parallel to the principal axis of symmetry as the cyclic number of that axis. These and the principal axis of symmetry are the only elements of symmetry, except in the tetra-trigonal class, where there are also three tetragonal inverse axes (which are, of course, also digonal simple axes) at right angles to one another. There are six planes of symmetry, and each of the four trigonal axes has three of these passing through it. In the Du row $n=2k$.

There is a close analogy between the Mh and Du rows. Every lateral digonal axis in the former is represented in the latter by the normal to a plane of symmetry, that is to say by a digonal inverse axis.

The dicyclic central, Dc, row combines the symmetry of the monocyclic holoaxial and the dicyclic uniterminal rows. As a result of the presence of the centre of symmetry, every line of symmetry, that is every even axis of symmetry, has a plane of symmetry at right angles to it. The crystals of the classes in this row are sometimes said to be holohedral, as they have the maximum symmetry and number of faces in the general form of any of the classes of the column and

* It is sometimes referred to as helical, because, although it has no planes of symmetry parallel to it, the terminations, like those of a screw, can be brought by rotation about a normal to the axis into coincidence with one another.

system to which they belong. The name is extended to the monoclinic central class, as that has the highest symmetry of the monoclinic system, and to the triclinic central class for a similar reason. In the Dc row $n=4k$.

The dicyclic inverse, Dv, row is similar to the monocyclic inverse row, but it has a number of planes of symmetry parallel to the principal axis equal to its cyclic number, if it be considered as a simple axis (half its cyclic number as an inverse axis), in addition to the plane of symmetry at right angles to the principal axis which is present both in the hexagonal and dihexagonal inverse classes (VIMv and VIDv). There are also the same number of digonal axes at right angles to the principal axis. In the ditetragonal inverse class these bisect the dihedral angles between the planes of symmetry parallel to the principal axis, while in the dihexagonal inverse class they lie in those planes. These digonal axes are in addition to the principal axis, which in both the tetragonal and ditetragonal inverse classes (IVMv and IVDv) is a simple digonal axis as well as an inverse tetragonal axis. In the Dv row $n=2k$.

As the principal axis in the hexagonal inverse (VIMv) class and the dihexagonal inverse class (VIDv) is a trigonal simple axis, these classes are usually placed in the trigonal system, but the principal axis is in fact an hexagonal inverse axis, and their proper place is in the hexagonal system, as was first pointed out by Professor Hilton.*

The absence of crystals that can be referred to the monocyclic classes IVMv and VIMv, and the rarity of crystals crystallising in the other classes with low symmetry in each system, are probably not due to the absence of ultimate cells possessing such reduced symmetry, but to the fact that the outer characters of such cells are so similar that they can be employed indiscriminately in building up a crystal, so that the latter is of a more generalised type, and therefore possesses higher symmetry than each individual cell.

In some cases the outer form appears to indicate a

* *Min. Mag.*, vol. xiv., pp. 261-263 (1907).

higher symmetry than that which the structure as a whole possesses, but evidence of the lower symmetry of the latter is afforded by the variation in the action of solvents along different directions, which results in the formation of what are known as etched figures on the surface of the faces, or by the phenomena of X-rays reflection, or by other physical characters mainly optical or electrical.

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